Problem Session #1

0. .... ChatGPT ✓

1. general solution strategies

2. Euler-Bernoulli beam theory

3. (potentially) Voigt notation

Assumptions: { quasi-static
infinitesimal deformations.
isotropic assumptions.

Overall {
① define problem (geometry. B.C.s).
② governing equations. (equilibrium, compatibility)
③. constitutive laws. (materials)
④. solution method. (FEM, stress function.)
FFT

Step #1

disp. strain stress.

$u_j \longleftrightarrow \varepsilon_{ij} \longleftrightarrow \sigma_{ij}$

3×1 : 3×3 : 3×3

kinematics Consti. laws.

$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$

linear elas.

$\sigma_{ij} = C_{ijkl}\varepsilon_{kl}$

traction. T_j

stress. σ_{ij}

$T_j = \sigma_{ij} n_i$

Step #2. Governing equations. (ALWAYS satisfied!)

- Equilibrium equation. $\sigma_{ij,i} + F_j = 0$.

$\sigma_{11,1} + F_1 = 0$
$\sigma_{21,2} + F_2 = 0$
$\sigma_{31,3} + F_3 = 0$

- Compatibility. $\varepsilon_{ij,kl} + \varepsilon_{kl,ij} - \varepsilon_{ik,jl} - \varepsilon_{jl,ik} = 0$

$\varepsilon_{11,11} + \varepsilon_{11,11} - \varepsilon_{11,11} - \varepsilon_{11,11} = 0$.

Step #3 Constitutive laws.

... depends on materials, loading rate, temperature, electromagnetic, (multiphysical) ...

generalized Hooke's law. Symmetry

$\sigma_{ij} = C_{ijkl}\varepsilon_{kl}$

major: $C_{ijkl} = C_{klij}$
minor: $C_{ijkl} = C_{jikl}$

isotropic

$C_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$

$C_{1111} = \lambda + 2\mu$

$C_{1122} = \lambda$

$\sigma_{ij} = \lambda\delta_{ij}\varepsilon_{kk} + 2\mu\varepsilon_{ij}$

...on #1

✓

...tion strategies

...lli beam theory

...) Voigt notation

...uasi-static

...nfinitesimal deformations.

...isotropic assumptions.

... problem (geometry B.C.s).

...g equations. (equilibrium, compatibility)

...tutive laws. (materials)

... method. (FEM, FEM, stress function.)

Step #1.

disp. strain stress.

$u_j \longleftrightarrow \varepsilon_{ij} \longleftrightarrow \sigma_{ij}$

9×1 ⋮ 3×3 ⋮ 3×3

kinematics Consti. laws.

$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$

↓ linear elas.

traction. T_j

↑

stress.

$\sigma_{ij} = C_{ijkl}\varepsilon_{kl}$

σ_{ij} T_j

$T_j = \sigma_{ij} n_i$

Step #2. Governing equations. (ALWAYS satisfied!)

- equilibrium equation. $\sigma_{ij,i} + F_j = 0$ ← body force

$\sigma_{i1,i} + F_1 = 0$ ... x

$\sigma_{i2,i} + F_2 = 0$ ... y,

$\sigma_{i3,i} + F_3 = 0$ ... z

- compatibility. $\varepsilon_{ij,kl} + \varepsilon_{kl,ij} - \varepsilon_{ik,jl} - \varepsilon_{jl,ik} = 0$

$\varepsilon_{11,11} + \varepsilon_{11,11} - \varepsilon_{11,11} - \varepsilon_{11,11} = 0.$

Step #3 Constitutive laws.

... depends on materials, loading rates, temperature electromagnetics, (multiphysics) ...

Generalized Hooke's law. Symmetries

$\sigma_{ij} = C_{ijkl}\varepsilon_{kl}$

major: $C_{ijkl} = C_{klij}$

minor: $C_{ijkl} = C_{ijlk}$

isotropic

$C_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$

$C_{1111} = \lambda + 2\mu$

$C_{1112} = 0$

$\sigma_{ij} = \lambda\varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij}$

$\varepsilon_{ij} = \frac{1}{E}\left(\sigma_{ij} - \nu(\sigma_{kk}\delta_{ij} - \sigma_{ij})\right)$

e.g., polymers, polycrystals, glass, ...

3×1

6×1

= [6×...]

$\underline{\underline{\sigma}} = \underset{\sim}{C}\,\varepsilon$ ...

$\sigma_{ij} = C_{ijk...}$

[3×3] = ...

$F_1 = 0$ ... x

$F_2 = 0$ ... y,

$F_3 = 0$ ... z

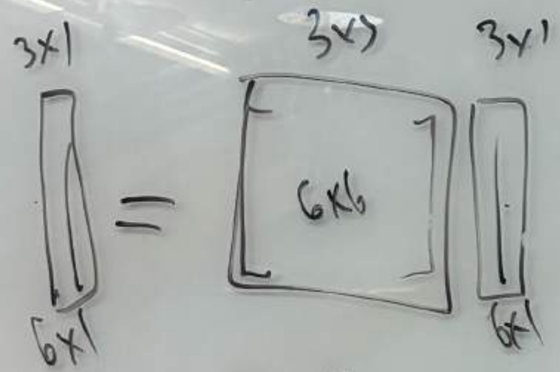
$\dots \delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$

2μ

$\lambda \varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij}$

$= \frac{1}{E}[\sigma_{ij} - \nu(\sigma_{kk}\delta_{ij} - \sigma_{ij})]$

e.g., polymers, polycrystals, glass, ...

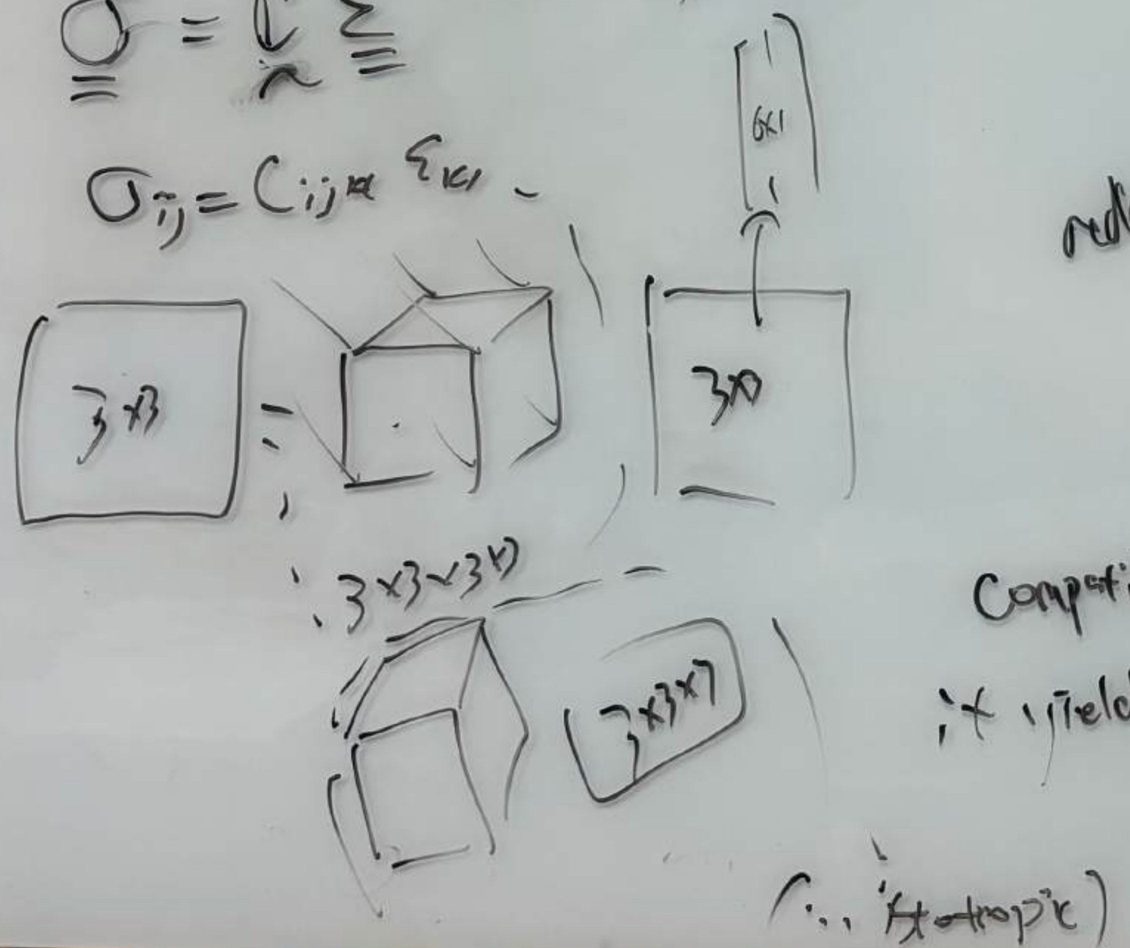


$\underline{v} = \underline{\underline{A}}\,\underline{u}$

$v_j = A_{ij} u_i$

$\underline{\underline{\sigma}} = \underset{\sim}{\mathbb{C}}\, \underline{\underline{\varepsilon}}$

$\sigma_{ij} = C_{ijkl}\varepsilon_{kl}$



Step #4 Solution method.

Stress function.

2D problems. $\sigma_{xx}(x,y)$, $\sigma_{yy}(x,y)$, $\sigma_{xy}(x,y)$

find all stress comp. at once.

reformulate in terms of $\phi(x,y)$

redefine:

$$\begin{cases} \sigma_{xx} = \phi_{,yy} \\ \sigma_{yy} = \phi_{,xx} \\ \sigma_{xy} = -\phi_{,xy} \end{cases}$$

(No body force).

... automatically satisfy equilibrium

recall step #1 & #2

Compatibility & BCs ... (what's left?).

it yields. $\nabla^4\phi = \nabla^2(\nabla^2\phi)$.

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\phi = 0$$