Problem Session #2

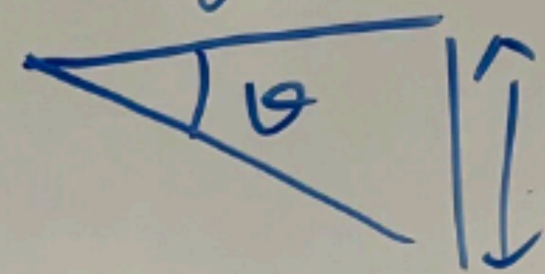
ME 80

- Euler-Bernoulli beam theory
- Stress function
  (code for HW #2)
  ⋮

lecture notes Pg. 16 onwards

Assumptions:
1. plane surfaces ($\perp$ to N.A.)
2. deflections are small.
3. No normal stress in N.A.

dx



dw

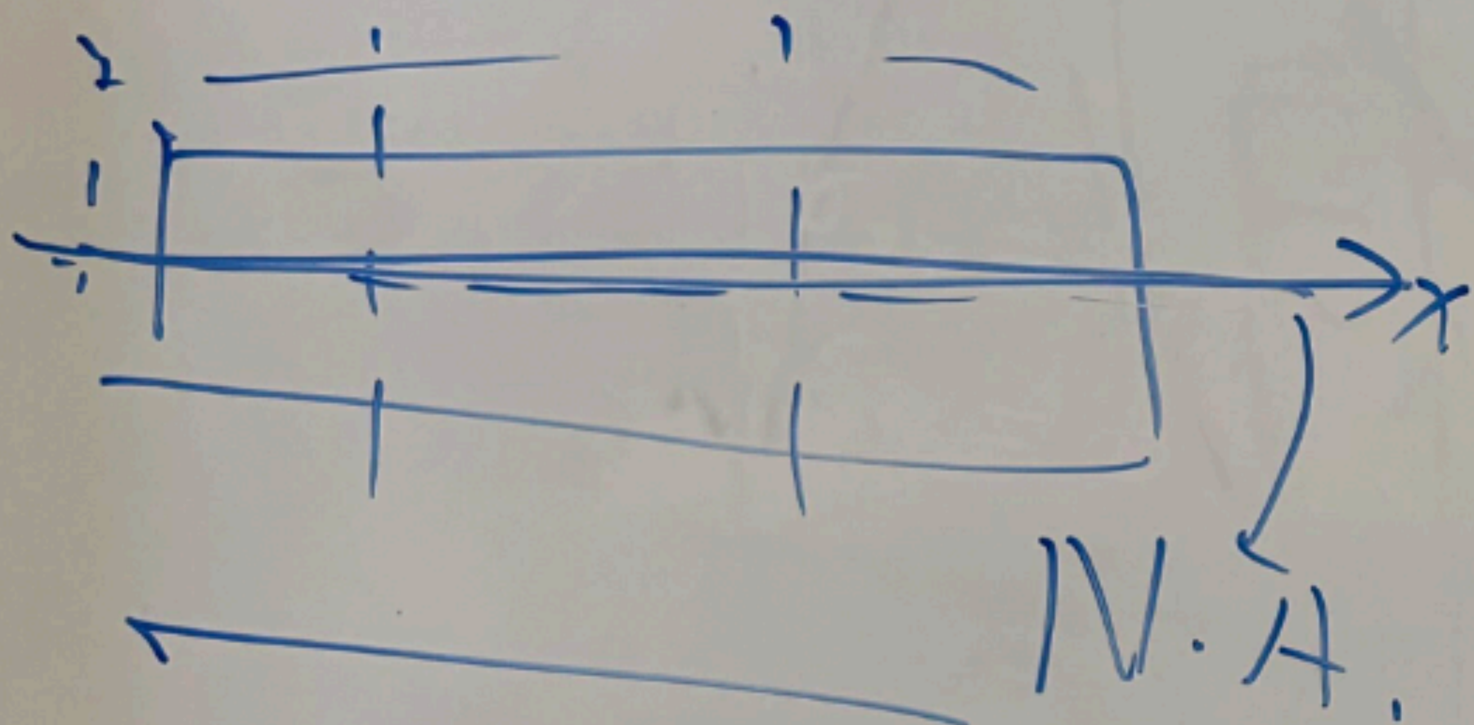
$\tan\theta \sim \theta = \frac{dw}{dx}$.

curvature (k) $= \frac{d\theta}{dx} = \frac{d^2w}{dx^2}$

$k = \frac{\text{bending moment}}{\text{resistance to bending}} = \frac{M}{k_s}$ → ince . .

k_s → EI

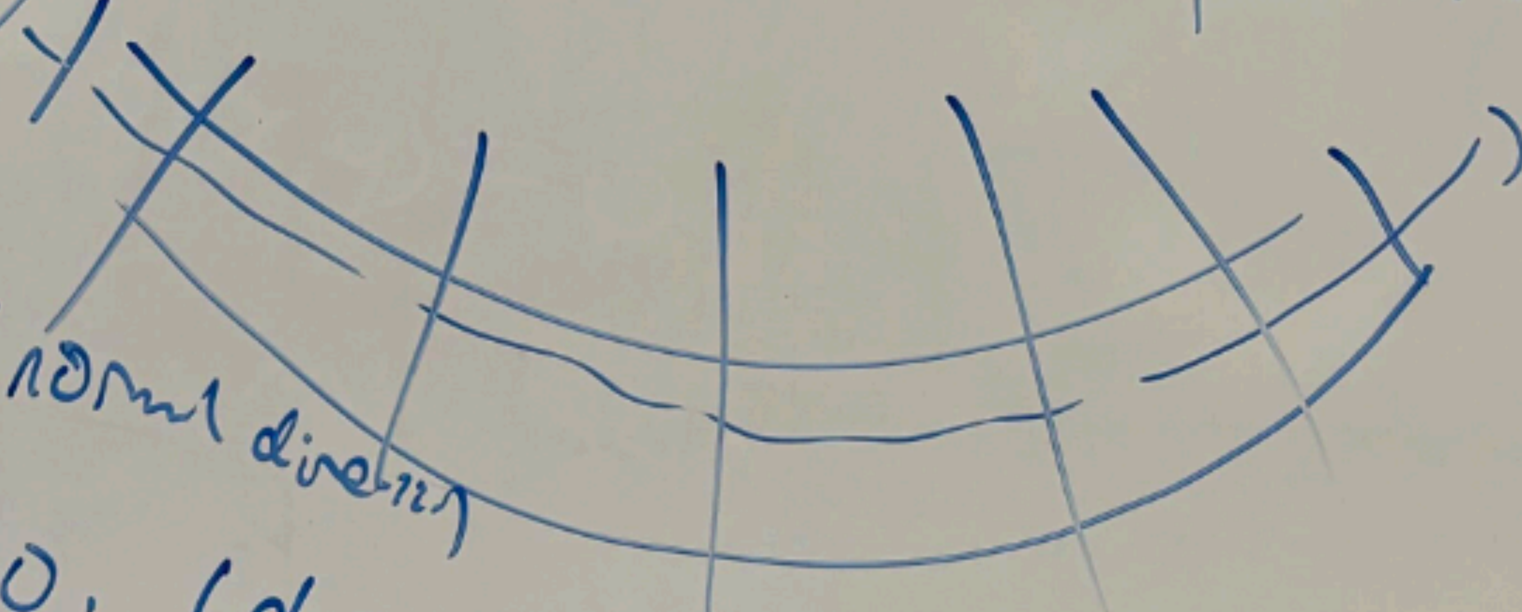
moment of inertia: $\int r^2 dm$.



N.A.

N.A. → axis (normal direction)

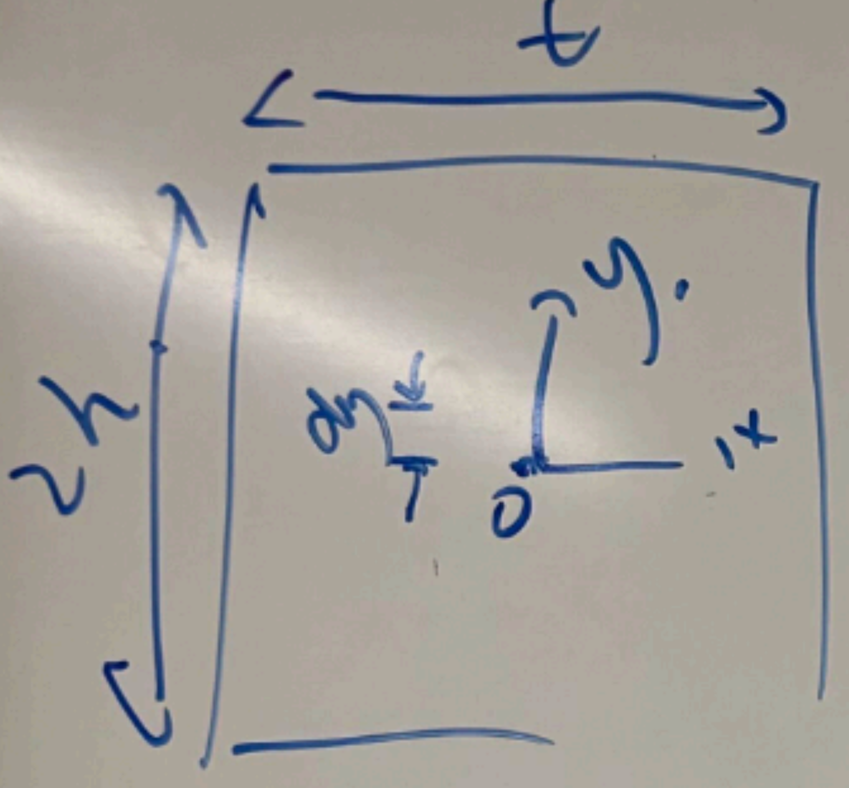
$\sigma_{xx} = 0$. (due to bending)



$M = EI_z \frac{d^2w}{dx^2}$.

Example #0

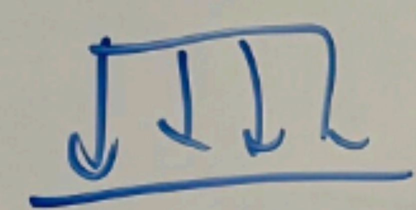
Second moment of area: $I_z = \int y^2 \, dA$.



$$I_z = \int_{-h}^{+h} y^2 t \, dy = \frac{2th^3}{3}$$

$$K_s = E I_z = \frac{2Eth^3}{3}$$

$$K = \frac{M}{EI_z} = \frac{d^2 w}{dx^2} \longrightarrow M = EI_z \frac{d^2 w}{dx^2}.$$



Governing equations:

balance of force. $\dfrac{dV}{dx} = -q(x)$.

balance of moment. $\dfrac{dM}{dx} = V(x)$

⋮

Equilibrium:

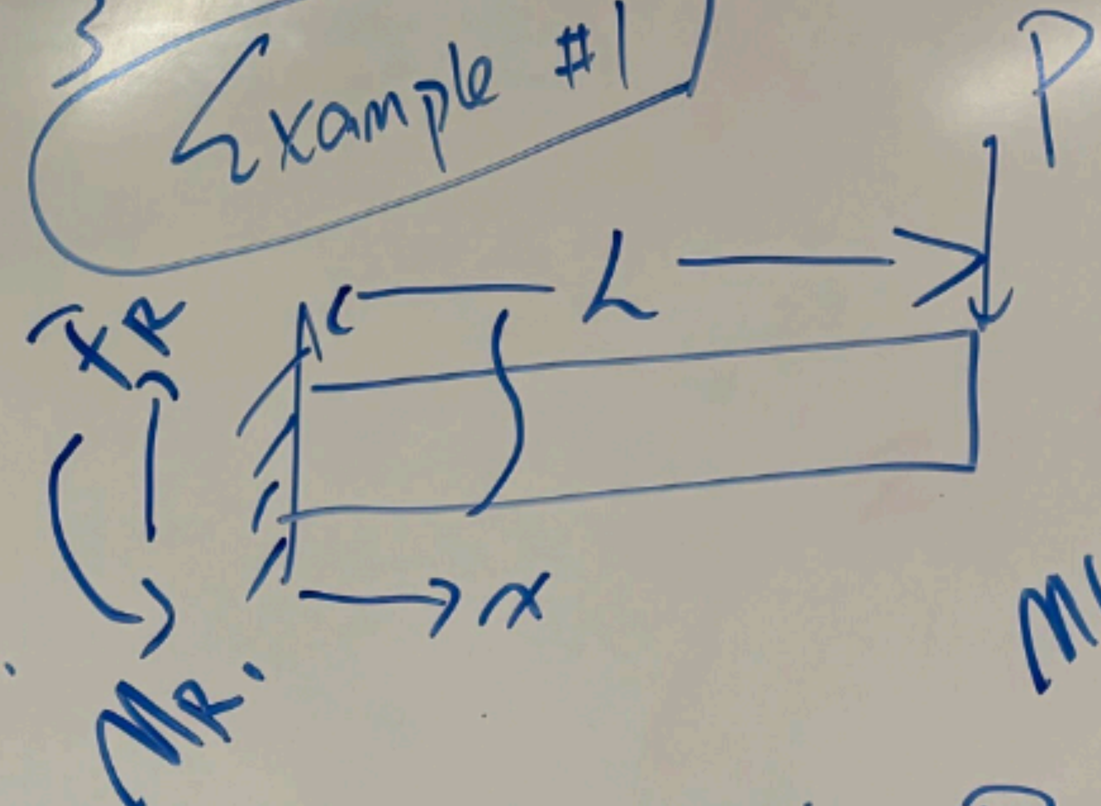
$$\cdots \quad q(x) = -EI_z \frac{d^4 w}{dx^4}$$

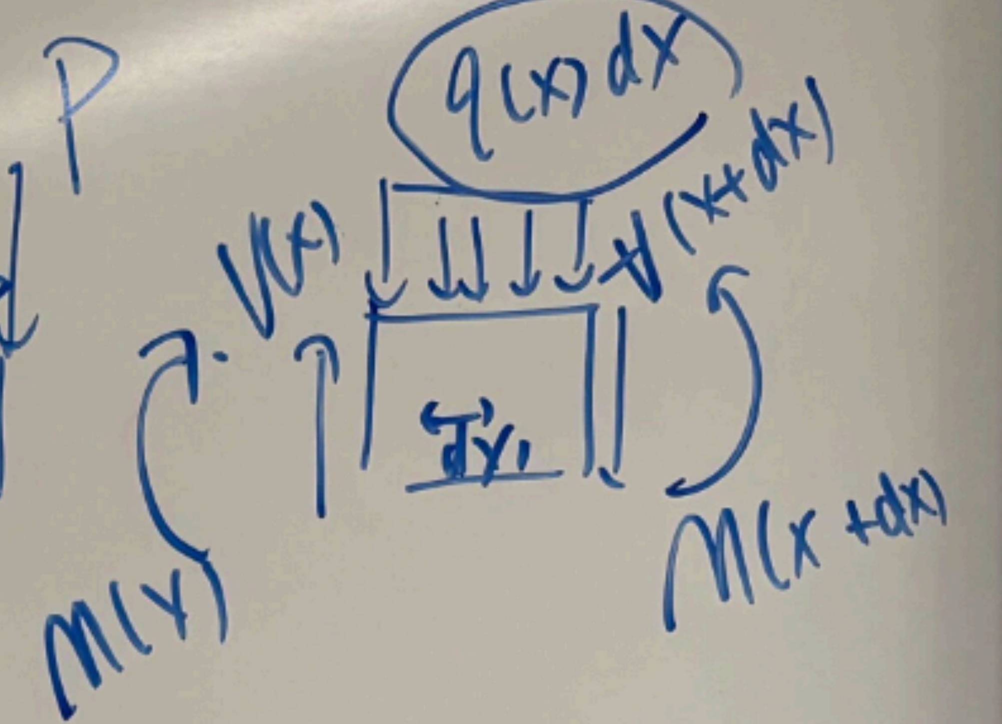
B.C.s? 4.

What do we want?

$V(x)$. $M(x)$.

Example #1

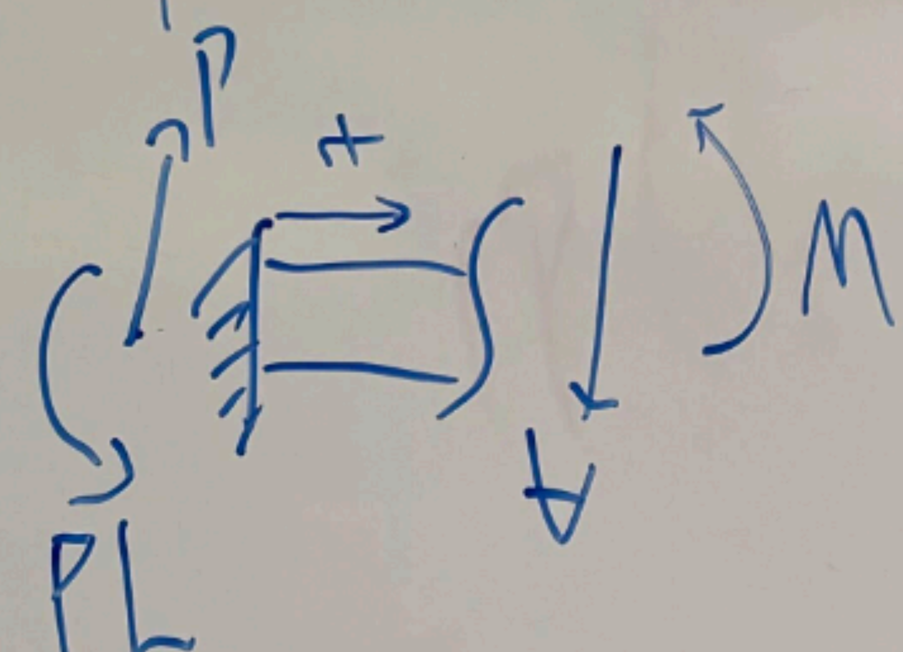


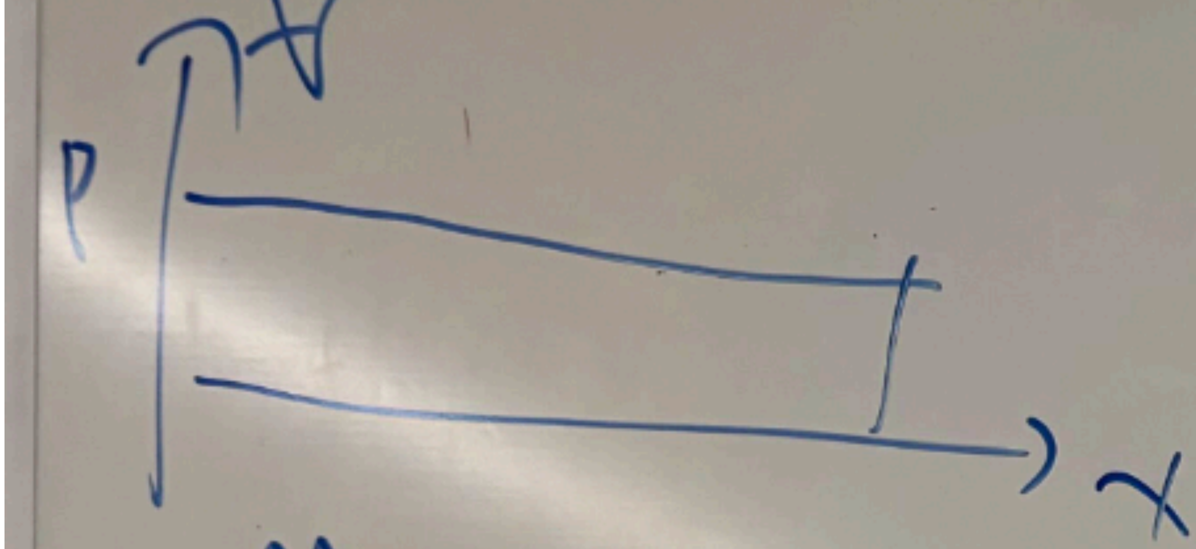


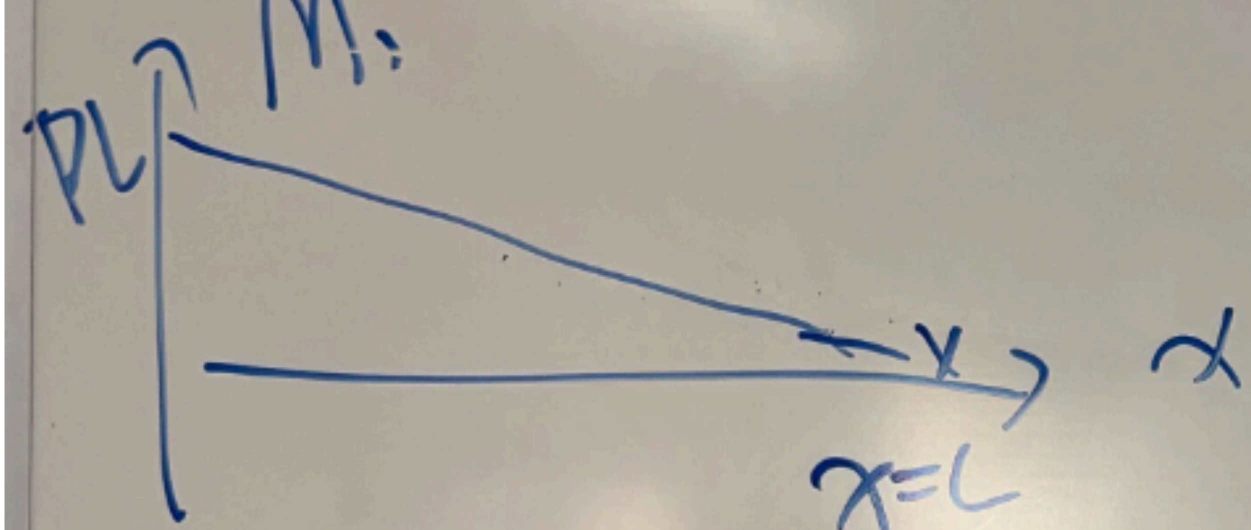
$\sum F = 0$: $V = P$.

$\sum M = 0$: $M + PL = Vx$.

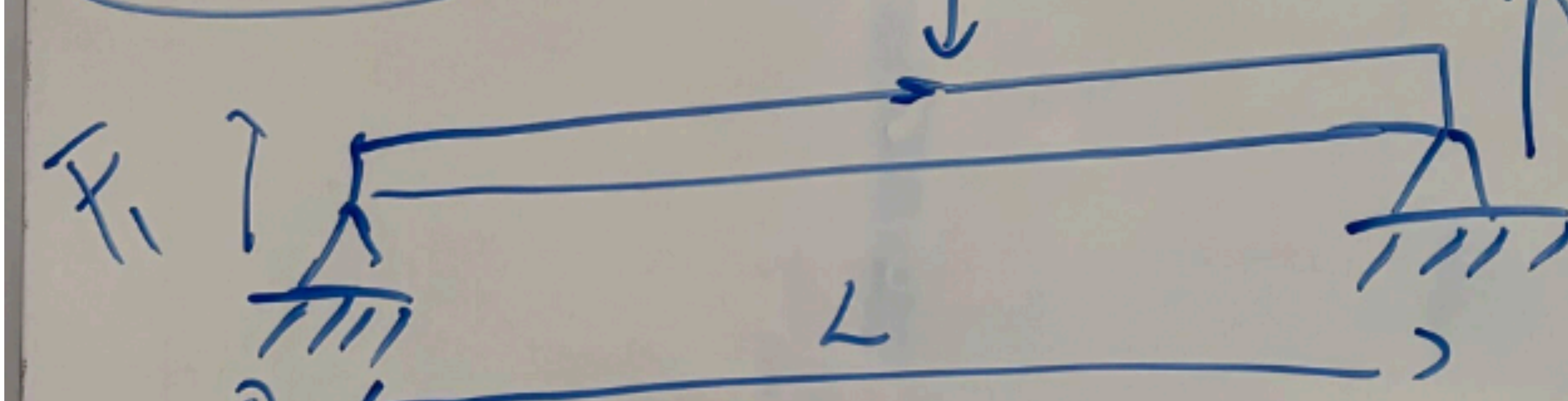
$\therefore M(x) = P(x - L)$







Example #2



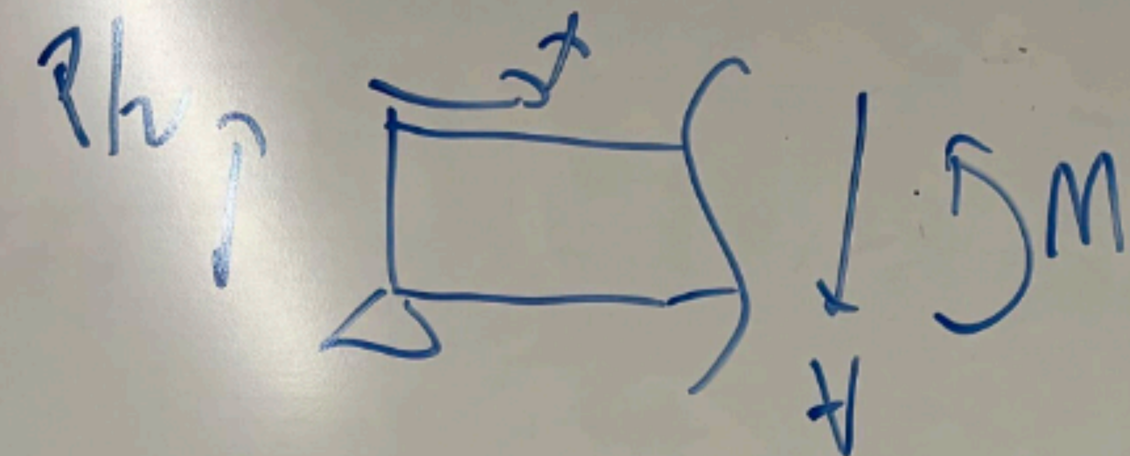
(freely supported ends, (no moment)

$F_1 = F_2 = P/2$

*** ONLY provide reaction force

DO NOT resist bending

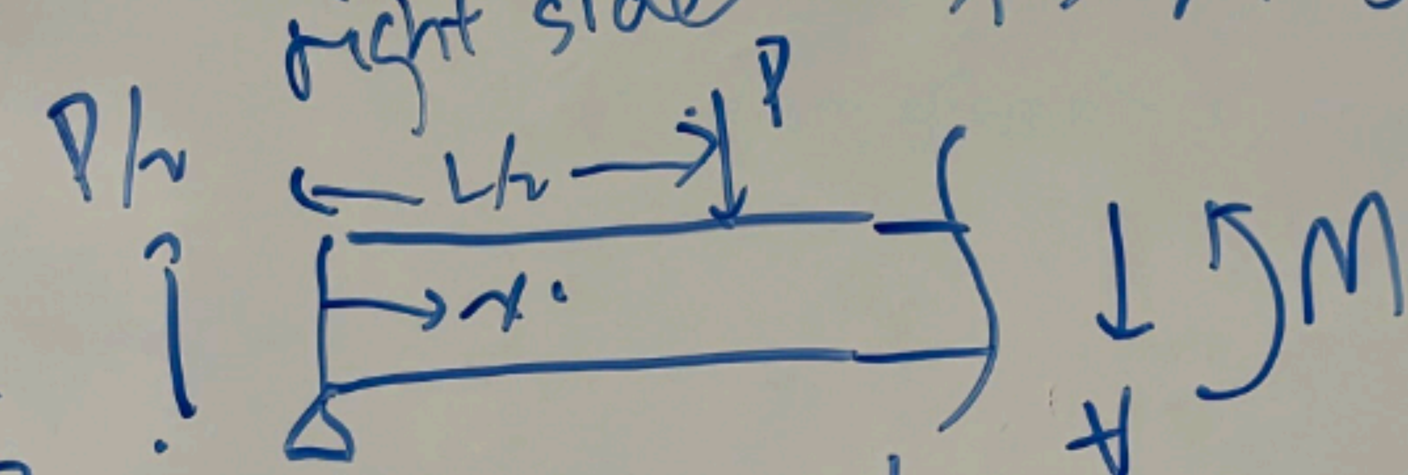
left side. $x < L/2$



$\sum F = 0: \quad V = P/2$

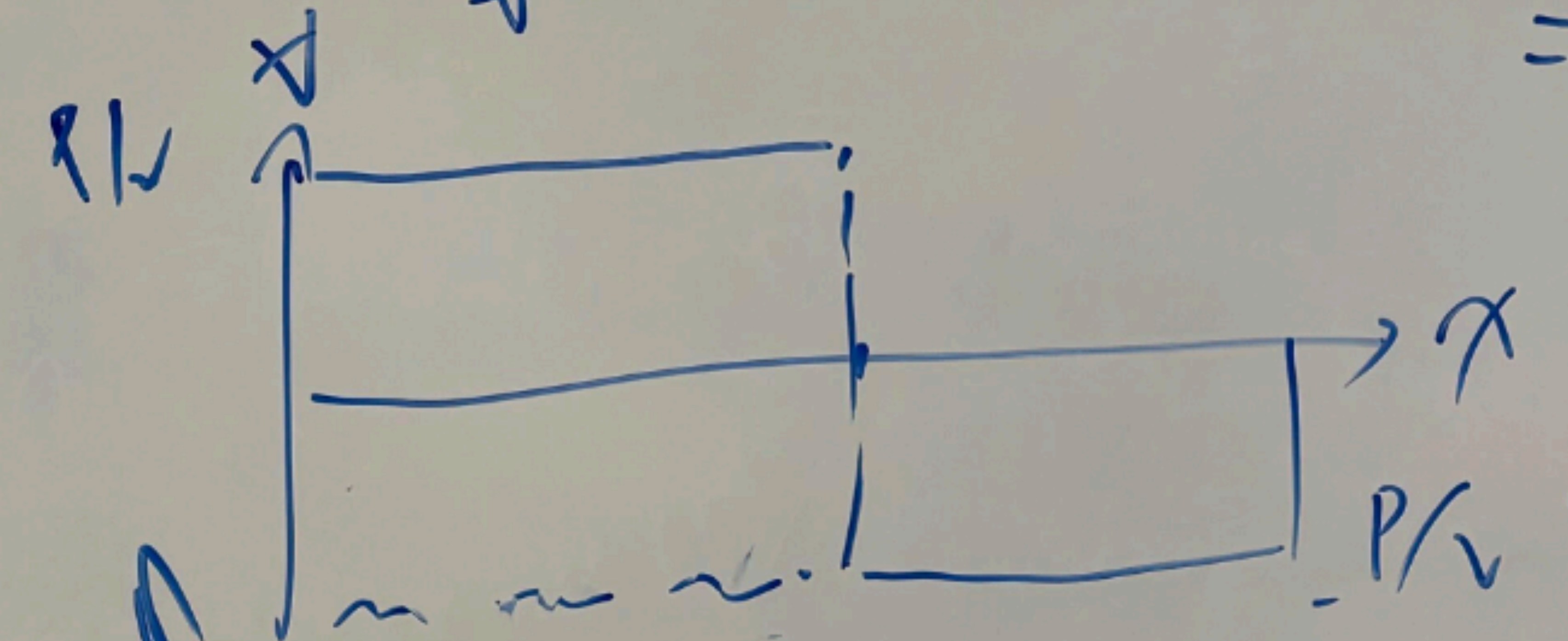
$\sum M = 0: \quad M(x) = V x = Px/2.$

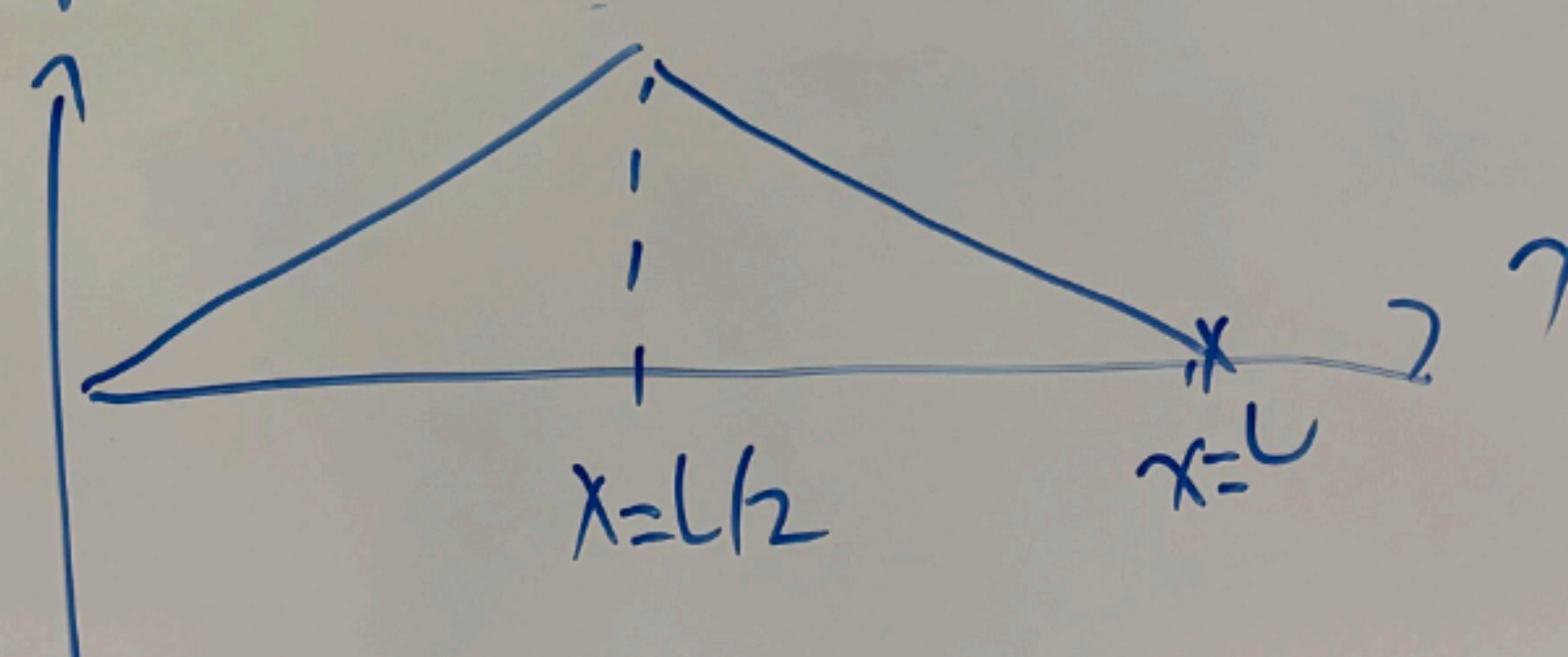
right side $x > L/2$



$\sum F = 0. \quad V = -P/2$

$$\sum M = 0: \quad M(x) = \frac{PL}{2} + Vx.$$
$$= P(L-x)/2$$





Stress function approach

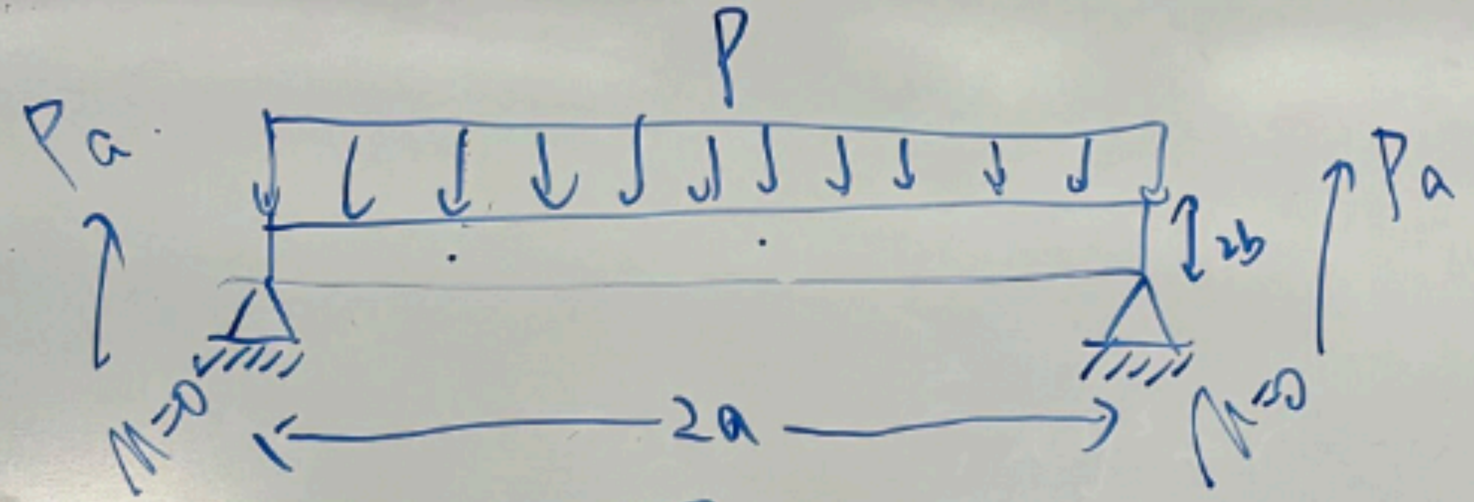
Lecture note #5 Pg. 11 ~ .

- Solve problems more accurately "probably".

- NOT exact .. – weak B.C.s

$\boxed{\text{HW \#2}}$ files/Homework/5522a.m

Barber

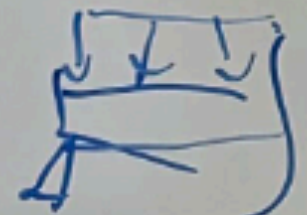


$q(x) \propto x^{0 \cdots n}$

$V(x) \propto x^1$

$M(x) \propto x^2$

$\sigma_{xx} \propto M(x) \cdot y \rightarrow \phi \propto x^{n+5}$



generalized Hooke's law, $\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl}$

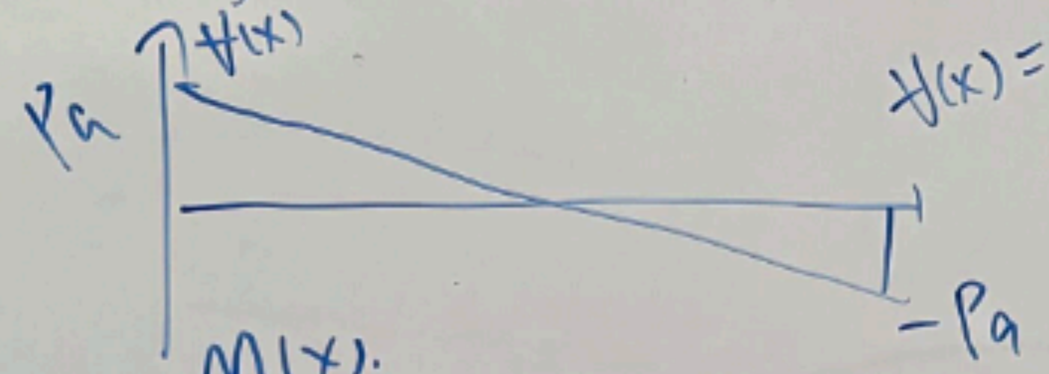
Governing eqn.

$\nabla^4 \phi = 0.$

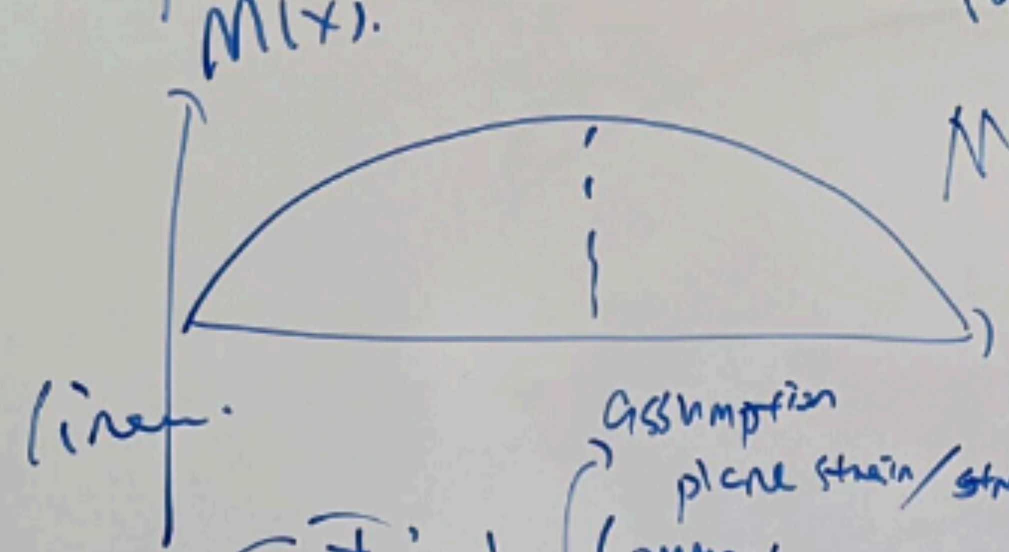
$= C_1 x^2 + C_2 x + \cdots + C_8 y^5.$

$u \leftarrow \underline{\underline{\varepsilon}}$ deformation

Using Euler-Bernoulli beam theory.



$V(x) = -Px.$



$M(x) = \frac{1}{2}P(a^2 - x^2).$

$\sigma_{xx}(x,y) = -\frac{M(x)y}{I_z}$

$= \frac{3P}{4b^3}(x^2 - a^2)y$

linear.

assumption

plane strain/stress

Comment

Find

$\sigma_{xx}(x,y) = \ldots$ $\varepsilon_{xx}(x,y) \ldots$

$\sigma_{yy}(x,y) = \ldots$ $\rightarrow \varepsilon_{yy}(x,y) \ldots$

$\sigma_{xy}(x,y) = \ldots$ $\varepsilon_{xy}(x,y)$

disp.

$\int \varepsilon \ldots$

$u_x(x,y)$

$u_y(x,y)$

$\phi = C_1 x^2 + C_2 xy + C_3 y^2 + \dots + C_{18} y^5$

$t_1 = \xi_1 x^3 + \xi_2 x^2 \dots$

$t_2 = \dots x^3 \dots$

$t_3 = \cdot x^3 \cdot$

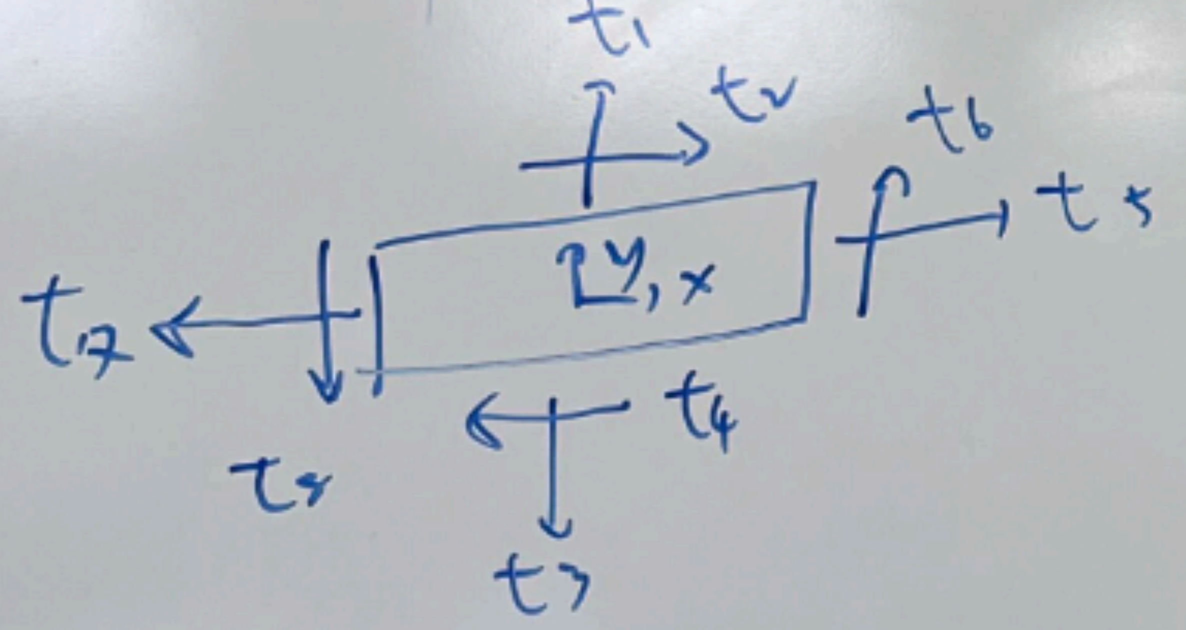
$t_4 = \dots x^3 \cdot$

Sxx : $\sigma_{xx} = \phi_{,yy}$

Syy : $\sigma_{yy} = \phi_{,xx}$

Sxy : $\sigma_{xy} = -\phi_{,xy}$

$T_j = \sigma_{ij} n_i$



t_1 : $\sigma_{yy}(x, y=b)$

t_2 : $\sigma_{xy}(x, y=b)$

t_3 : $\sigma_{yy}(x, y=-b)$

t_4 : $\sigma_{xy}(x, y=-b)$

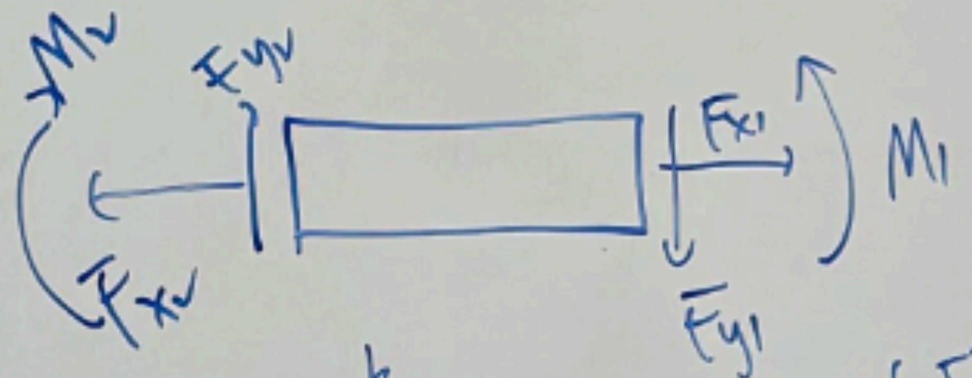
t_5 : $\sigma_{xx}(x=a, y)$

t_6 : $\sigma_{xy}(x=a, y)$

t_7 : $\sigma_{xx}(x=-a, y)$

t_8 : $\sigma_{xy}(x=-a, y)$

$\sigma_{ij} = \frac{\text{force } j}{\text{area } i}$



F_{x1}: $\int_{-b}^{b} t_5\, dy$

F_{y2}: $\int_{-b}^{b} t_6\, dy$

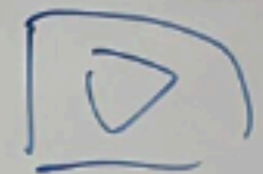
$M1$: $\int_{-b}^{b} t_5\, y\, dy$

F_{x2}: $\int_{-b}^{b} t_7\, dy$

F_{y2}: $\int_{-b}^{b} t_8\, dy$

$M2$: $\int_{-b}^{b} t_7\, y\, dy$

int(t5, [-b, b])



$S_1 = 0 \quad S_2 = 0 \quad S_3 = 0 \quad S_4 + p = 0$

$S_5 = 0 \quad S_6 = 0 \quad S_7 = 0 \quad S_8 = 0$

$S_9 = 0 \quad S_{10} = 0 \quad S_{11} = 0 \quad S_{12} = 0$

$S_{13} = 0 \quad S_{14} = 0 \quad S_{15} = 0 \quad S_{16} = 0$

$b_1 = 0 \quad b_2 = 0 \quad b_3 = 0$

$*_1$ $F_{x1} = 0 \quad M1 = 0 \quad F_{y1} - pa = 0$

$F_{x2} = 0 \quad M2 = 0 \quad F_{y2} + pa = 0$

$\phi(x,y) = -\frac{P}{4b^3}(12x^2 b^3 + \dots)$

$\sigma_{xx}(x,y) = -\frac{Py}{2b^3} \dots$

$\sigma_{yy}(x,y) = \dots$

$\sigma_{xy}(x,y) = \dots$