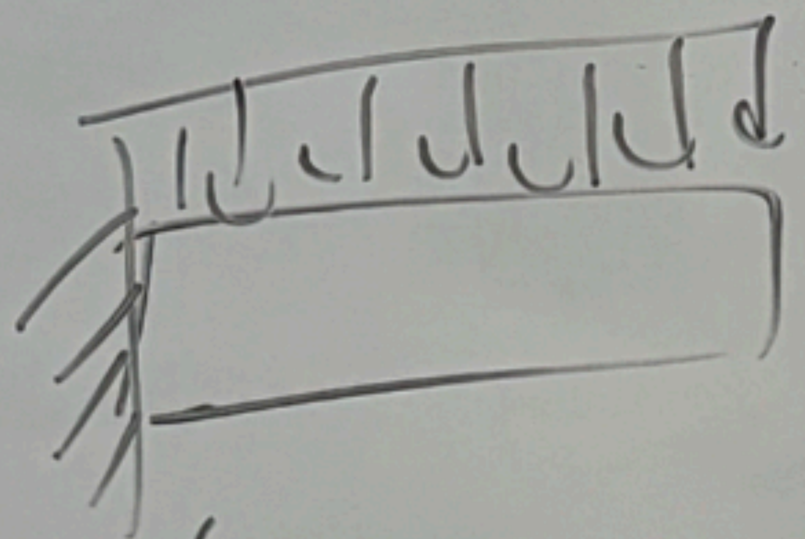
# Problem Session 4 Fri Jan 30 ME 340

## Goals

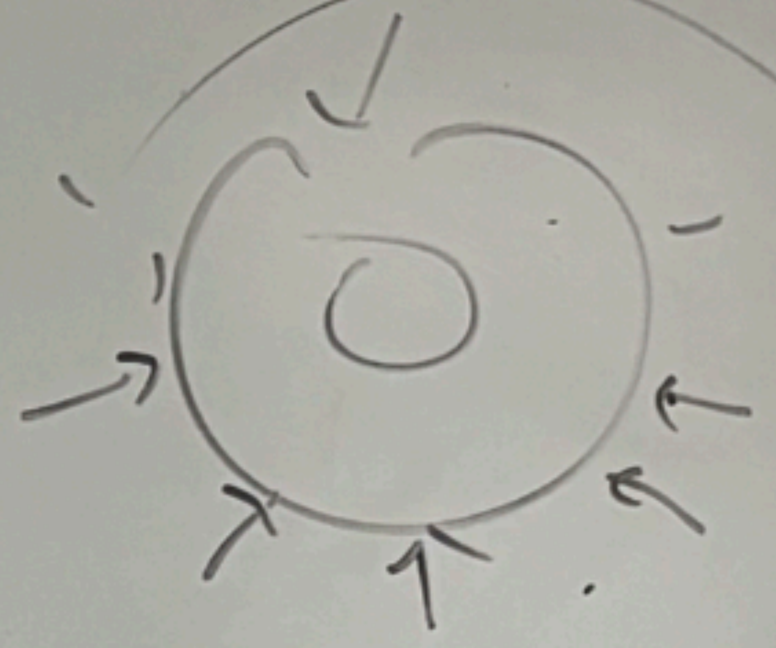
- Biharmonic equation in polar coordinates
- Michell solutions
- Example



$\phi(x,y) = \dots$

$r$ –

$\theta$ – ..



$\phi^{(0)} + \phi^{(1)}$

>> Lecture Notes / Barber Tables

>> Problem Session of ...

$$\nabla^2\phi = (m^2-n^2)\, r^{m-2} e^{in\theta}$$

$$\sqrt{(m-n)^2} = \sqrt{n^2}$$

$$\nabla^2(\nabla^2\phi) = 0 \qquad \nabla^2\left(r^{m-2}e^{in\theta}\right) = 0 \longrightarrow (m-2)^2 - n^2 = 0$$

$$m = \{n, -n, n+2, 2-n\}$$

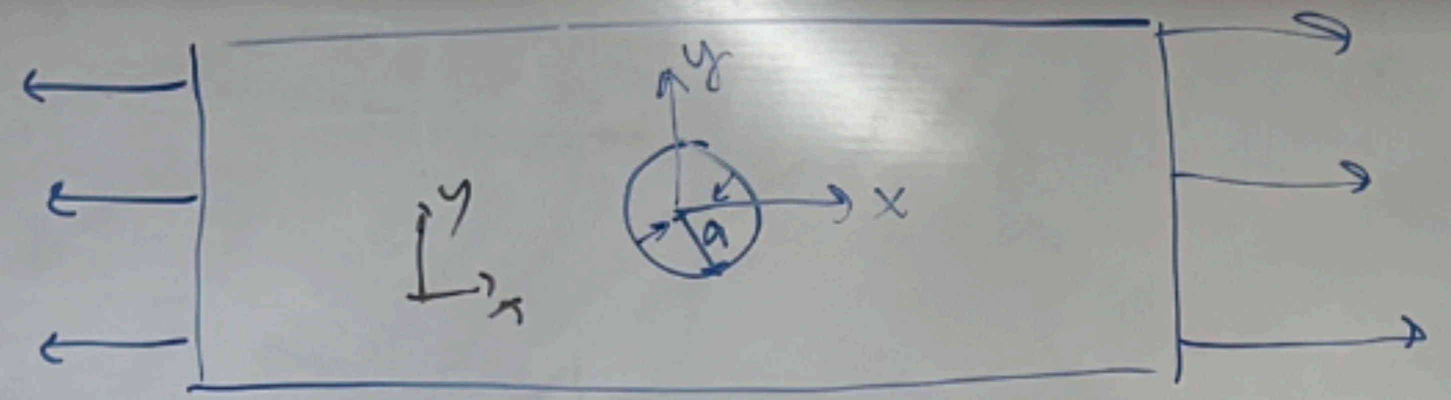
$$\phi_n(r,\theta) = \left(A_n r^{n+2} + B_n r^{2-n} + C_n r^n + D_n r^{-n}\right)\cos n\theta$$

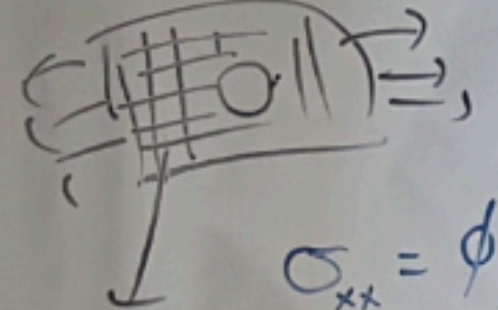
$$+ \left(\tilde{A}_n r^{n+2} + \tilde{B}_n r^{2-n} + C_n r^n + D_n r^{-n}\right)\sin(n\theta)$$

$$n = 0, 1 \qquad r^k \ln r$$

$$f(r) = r^m \ln r$$

$\sigma_{xx} = S$



$$\phi = \phi^{(0)} + \phi^{(1)}$$

$$\sigma_{xx} = \phi_{,yy} = S$$

K' = 2.3
2.5
2.7
2.9995


$$\phi^{(0)} = \frac{1}{2} S y^2 = \frac{S}{2} r^2 \sin^2\theta = \frac{S}{4} r^2 (1 - \cos 2\theta)$$

$$\sum_{n=0}^{\infty} f(r) e^{in\theta} = \sum_{n=0}^{\infty} A_n \cos n\theta$$

$$\sum_{n=0}^{\infty} A_n \cos n\theta$$

$\cos(0\theta) = 1$

$$\sigma_{rr}(a, \theta) = 0$$

$$\sigma_{r\theta}(a, \theta) = 0$$

$A_n \neq 0, \; n = 0, 2$

$A_n = 0$ else

13

$\sigma_{ij}$

r

Γ-surface

$$\int T_{ij} G_{ij}^{s}(x' - x)\, dx'$$

$$\phi^{(1)}(r,\theta) = A r^0 \ln r + \left(C + \frac{D}{r^2}\right)\cos 2\theta + B(\theta)$$

$$\sigma_{rr} = \sigma_{rr}^{(0)} + \sigma_{rr}^{(1)} = \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2}$$

$$= \frac{S}{2}\left(1 - \left(\frac{a}{r}\right)^2\right) + \frac{S}{2}\left(1 - 4\left(\frac{a}{r}\right)^2 + 3\left(\frac{a}{r}\right)^4\right)\cos 2\theta$$

$$\sigma_{\theta\theta}(r,\theta) = \frac{S}{2}\left(1 + \left(\frac{a}{r}\right)^2\right) - \frac{S}{2}\left(1 + 3\left(\frac{a}{r}\right)^4\right)\cos 2\theta$$

$$\sigma_{r\theta}(r,\theta) = -\frac{S}{2}\left(1 + 2\left(\frac{a}{r}\right)^2 - 3\left(\frac{a}{r}\right)^4\right)\sin 2\theta$$

1. $\phi(r,$

$\phi$

$\sigma_{r\theta}$ $\sigma_{\theta\theta}$ $\sigma_r$

$u_r$ $u_\theta$

$u_\theta \quad u_r \quad \sigma_{rr} \ldots$

1. $$\phi(r,\theta) = A_0 r^2 + B_0 r^2 \ln r + C_0 \ln(r)$$

$$+ (I_0 r^2 + I_1 r^2 \ln r + I_2 \ln r + I_3)\theta.$$

$$+ (A_1 r + B_1 r^{-1} + B_1' r\theta + C_1 r^3 + D_1 r \ln(r)) \cos\theta$$

$$+ (E_1 r + F_1 r^{-1} + F_1' r\theta + G r^3 + H_1 r \ln(r)) \sin\theta$$

$$+ \sum_{n=2}^{\infty} (A_n r^n + B_n r^{-n} + C_n r^{n+2} + D_n r^{2-n}) \cos(n\theta),$$

$$+ \sum_{n=2}^{\infty} (E_n r^n + F_n r^{-n} + G_n r^{n+2} + H_n r^{2-n}) \sin(n\theta)$$

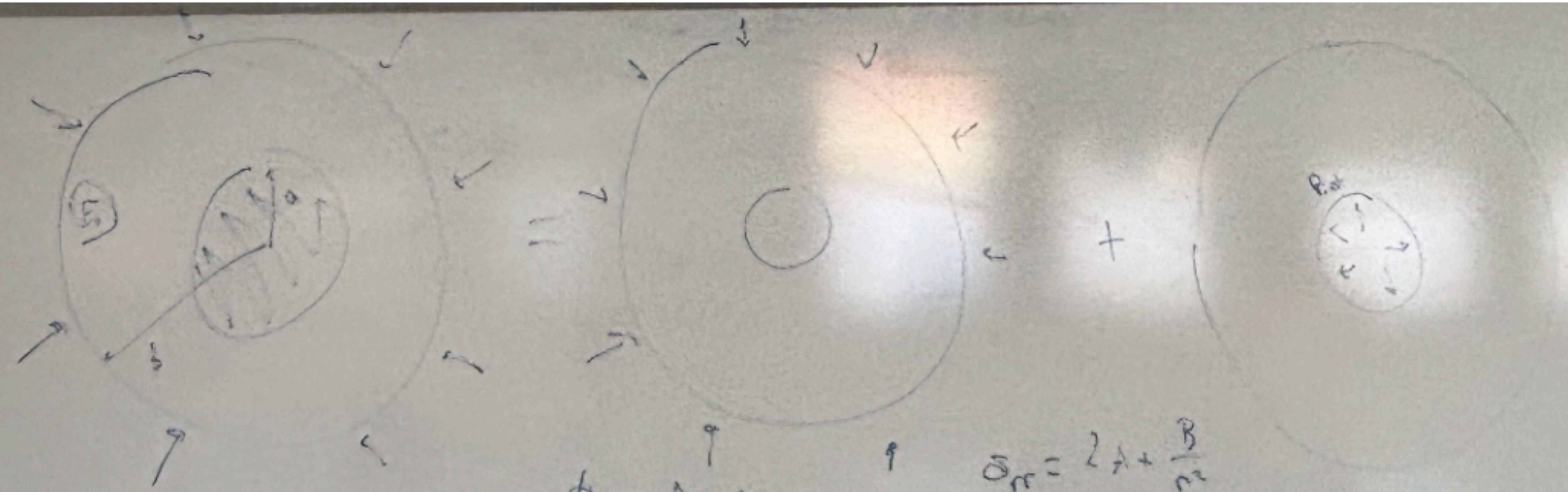
$\phi'$

$\phi$

$\sum$ ✓

$\sigma_{r\theta} \quad \sigma_{\theta\theta} \quad \sigma_{rr} \qquad n=1$

$u_r \quad u_\theta \qquad k$



$$2\mu u_r = A(k-1)r - \frac{B}{r}$$

$$A = \frac{-E_1}{2+(k-1)\frac{a^2}{b^2}}$$

$$B = \frac{-P(k-1)a^2}{2+(k-1)\frac{a^2}{b^2}}$$

$$\phi = A r^2 + B \ln r$$

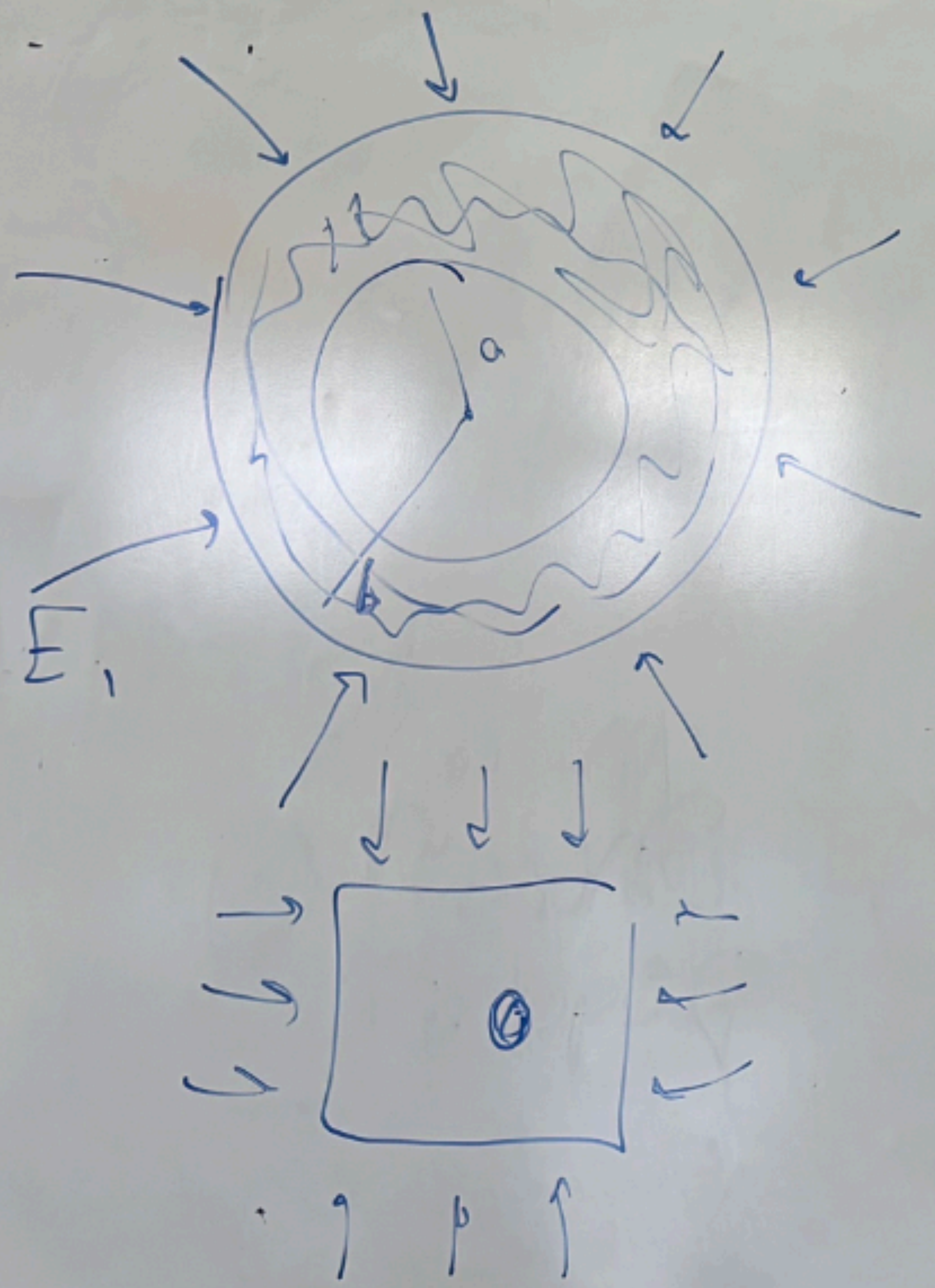
$$\sigma_{rr} = 2A + \frac{B}{r^2}$$

$$\sigma_{r\theta} = 0$$

$$\sigma_{\theta\theta} = E_1\left(2A - \frac{B}{r^2}\right)$$

$$\sigma_{rr}(r=b) = -E_1$$

$$u_r(r=a) = 0$$




$$\phi = \phi(r) = \{ r^2, \ln r, \cancel{r^2 \ln r} \}$$

$$\phi = r^2 \ln r \rightarrow u_\theta = (k+1) r \theta$$

$$u_\theta(\theta) \neq u_\theta(\theta + 2\pi)$$

$$\phi = A r^2 + B \ln r$$

$$K = 3 - 4\nu$$

$$\sigma_{rr} = 2A + \frac{B}{r^2}$$

$$\sigma_{r\theta} = 0$$

$$\sigma_{\theta\theta} = 2A - \frac{B}{r^2}$$

$$\sigma_{rr}(r = b) = -E_1$$

$$\sigma_{rr}(r = a) = 0$$

$$A = \frac{-E_1 b^2}{2(b^2 - a^2)} \qquad = 0$$

$$B = \frac{+E_1 a^2 b^2}{b^2 - a^2} \qquad = E_1 a^2$$

limiting case: $b \rightarrow \infty, \frac{a}{b} \rightarrow 0$

$$\sigma_{rr} = -E_1 a^2 / r^2$$

$$\sigma_{\theta\theta} = E_1 a^2 / r^2$$