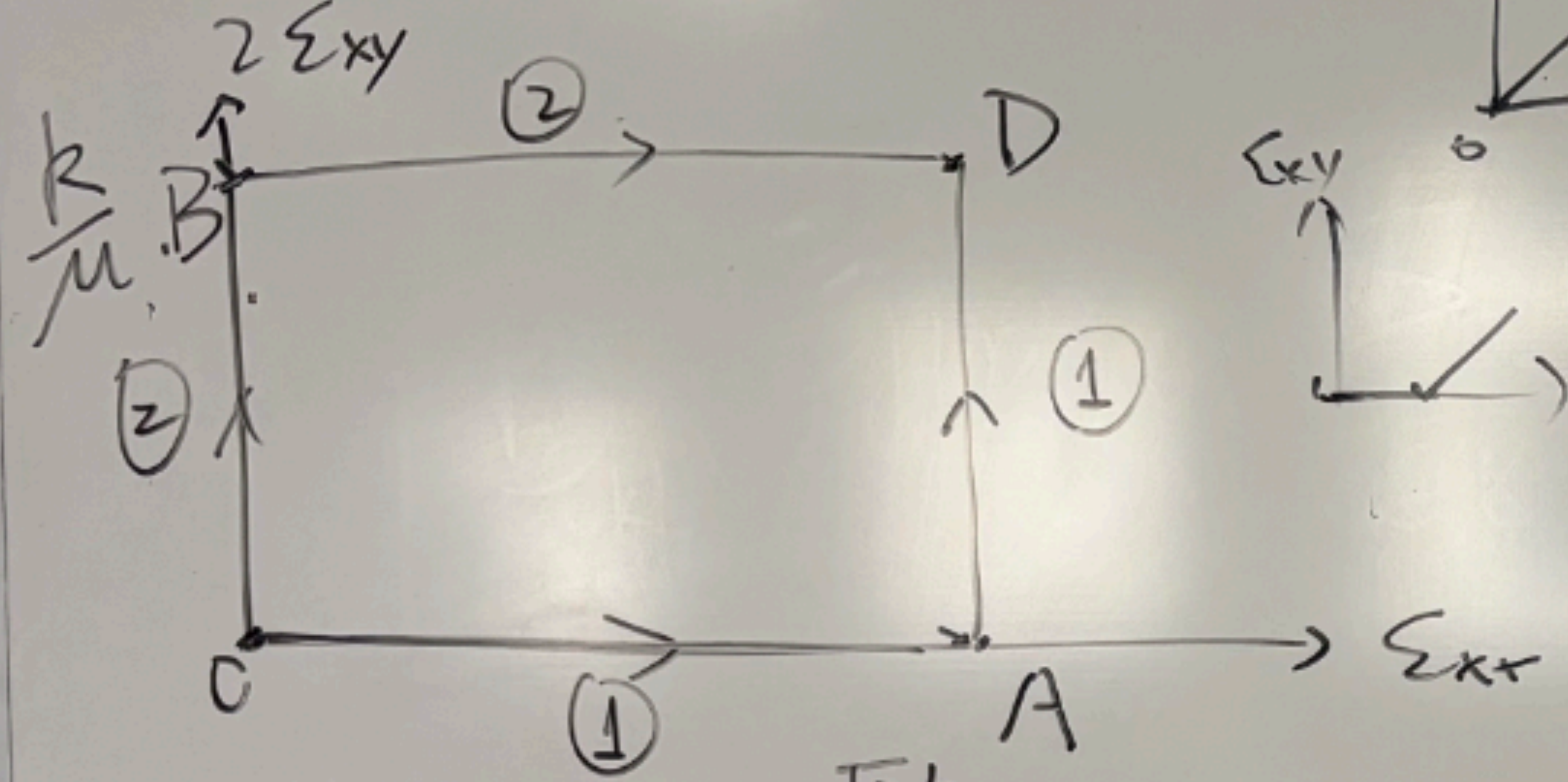
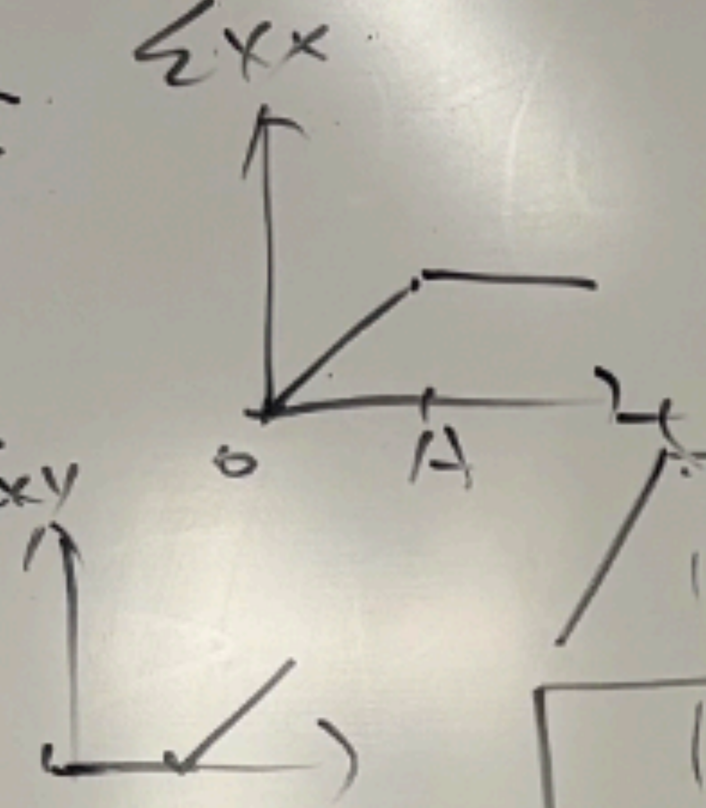
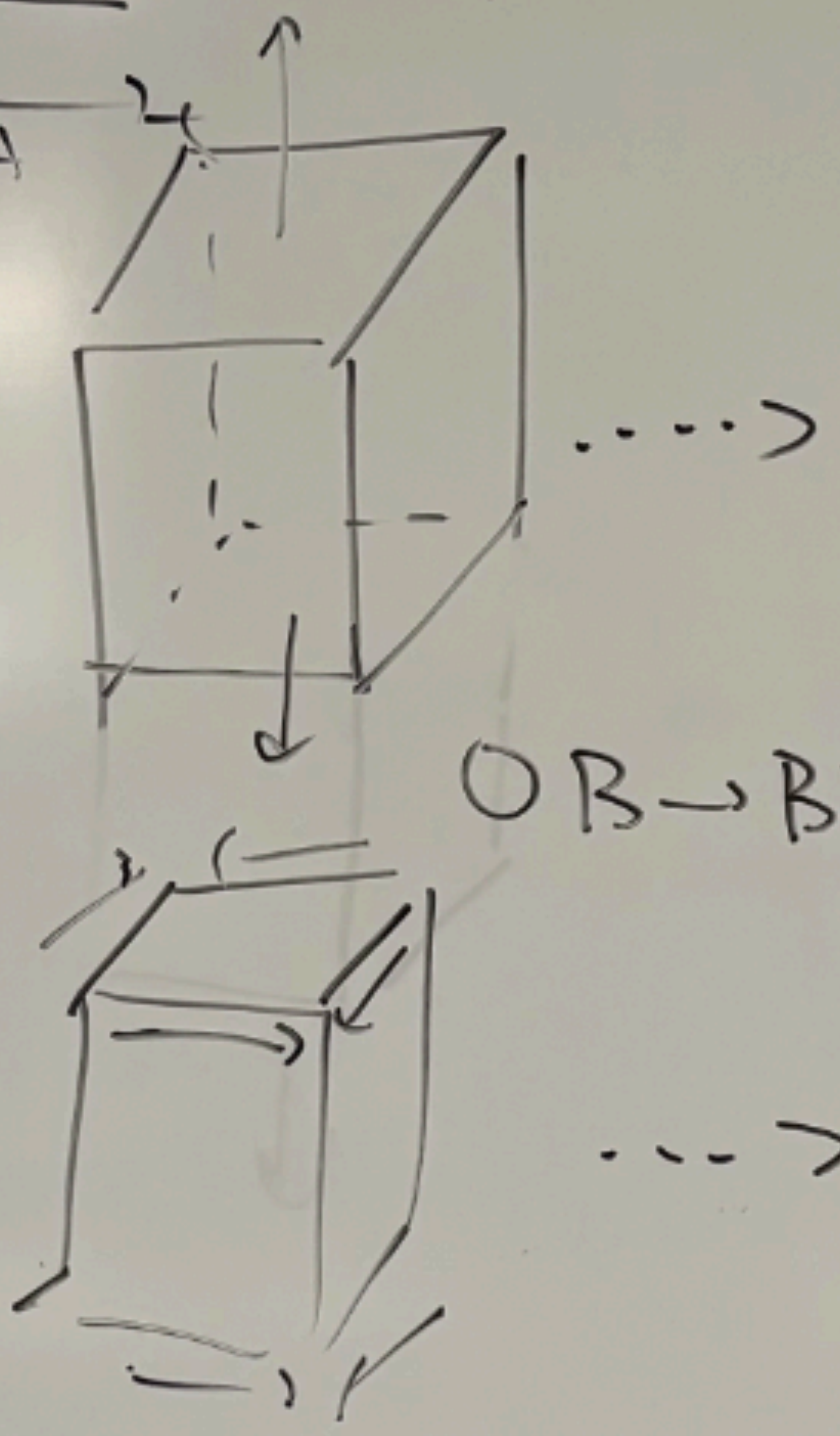
Problem Session

OA → AD. "first stretch it, then twist".

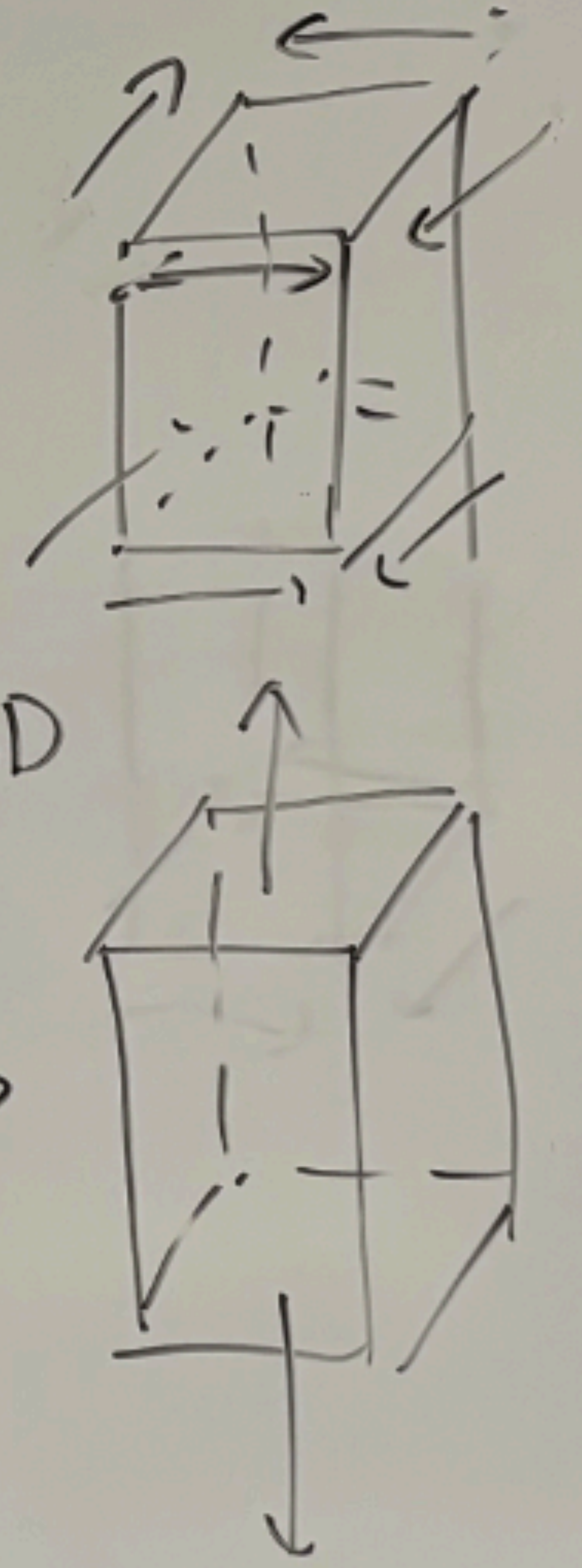


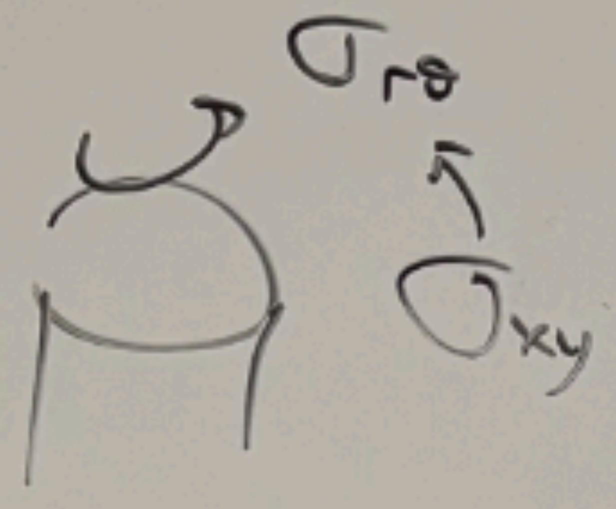


(a)

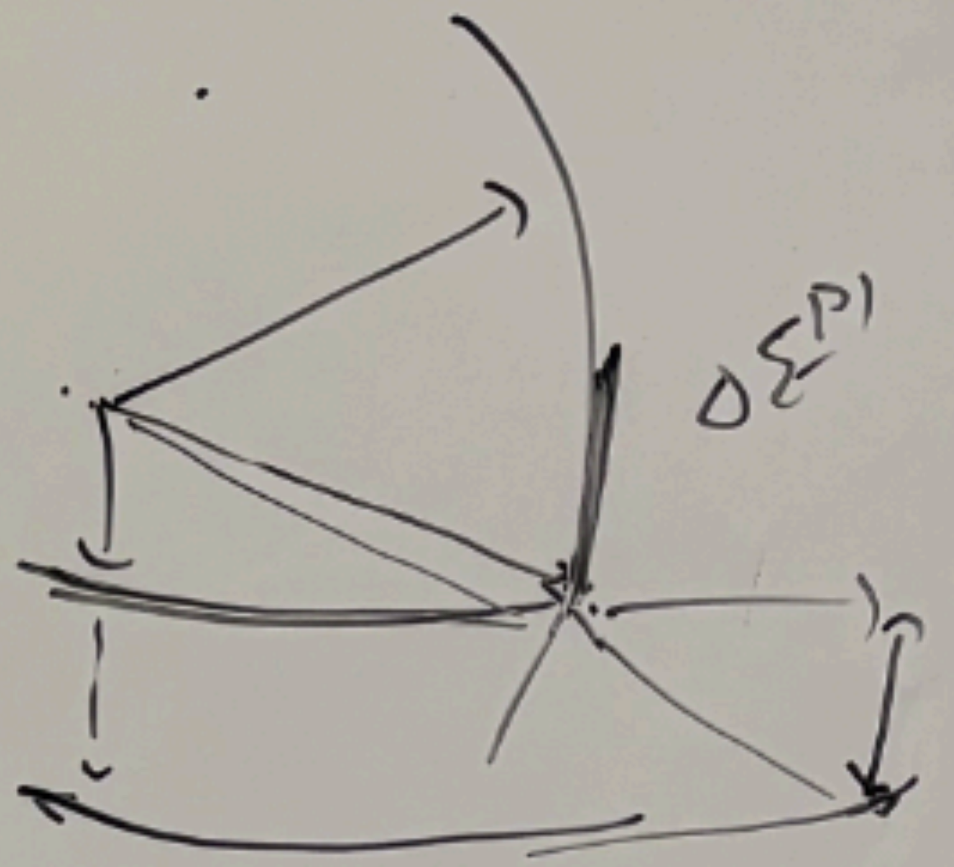


(b)





OB → BD



$\Delta\varepsilon^{pl}$ Δt

Incompressibility

$-\ \varepsilon^{el}_{ij} = e^{el}_{ij}$

$-\ \varepsilon^{pl}_{ij} = e^{pl}_{ij}$ ← deviatoric

$J_2 = k^2$ ...

$\frac{\sqrt{3}k}{E} \rightsquigarrow (\sigma_Y = \sqrt{3}k)$.

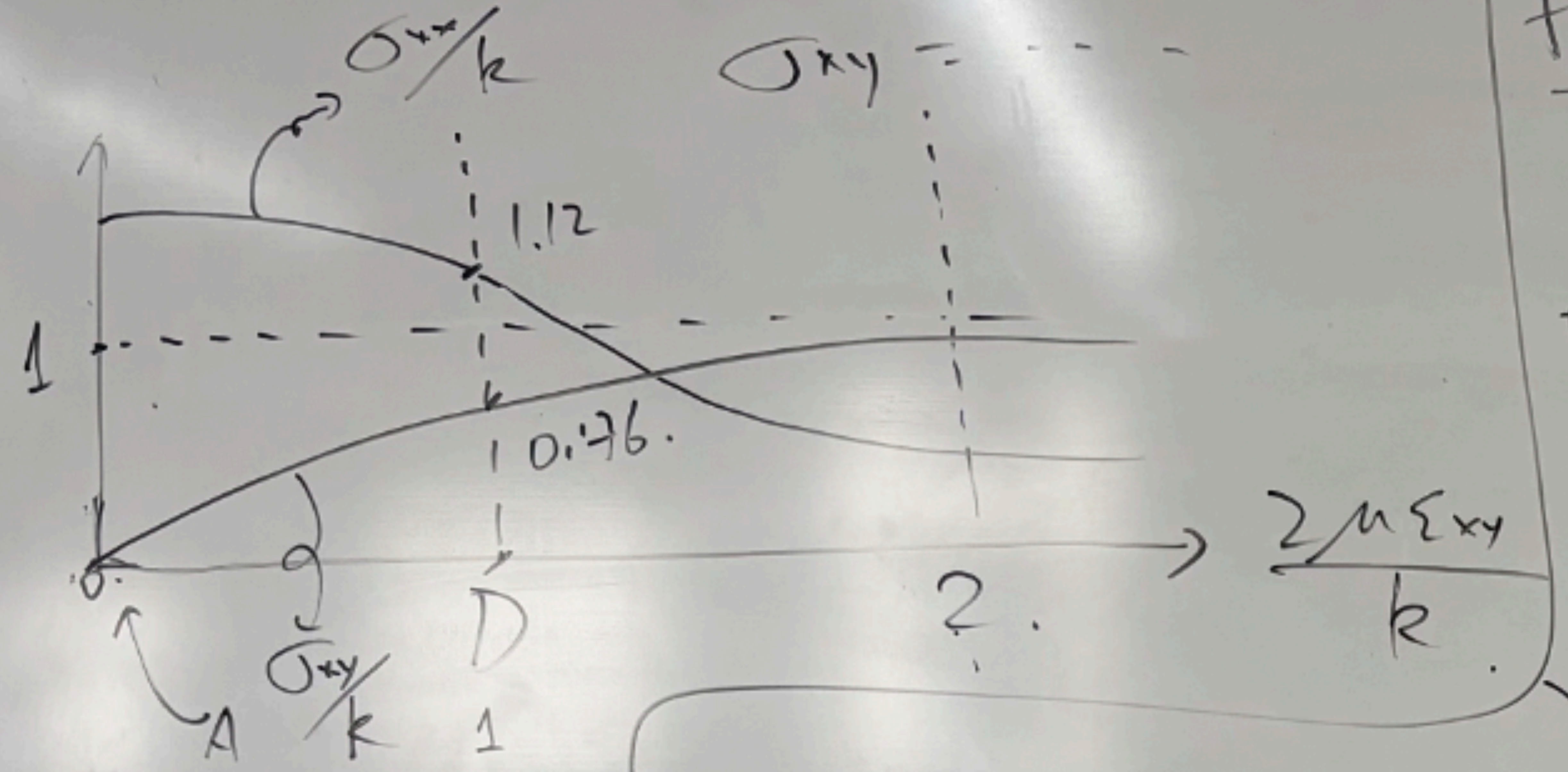
$E = 2\mu(1+\nu)$

$= 3\mu$

#1

$\sigma_{xx} = \ldots$

$\sigma_{xy} = \ldots$



$\sigma_{xx} = 1.12k$

$\sigma_{xy} = 0.76k$

OB → BD

OB: elastic

BD: plastic

$\sigma_{xy} = 2\mu\,\varepsilon_{xy}$

$\sigma_{xy} = \sigma_{yy} = \sigma_{zz} = \sigma_{yz} = \sigma_{zx} = 0$

$\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{zz} = \varepsilon_{yz} = \varepsilon_{zx} = 0.$

@ point B.

$\sigma_{xy} = k$

$\varepsilon_{xy} = k/2\mu$

*** for path BD

↳ material stays in the plastic flow regime.

plastic strain rate: $\dot{\varepsilon}^{pl}_{ij} = \dfrac{\dot{W}}{2k^2} S_{ij}.$

$\dot{W} = S_{ij}\,\dot{e}_{ij} \Longrightarrow$. rate of work associated with shape change

$\dot{\varepsilon}^{pl}_{xx} = \dfrac{\dot{W}}{2k^2} S_{xx} = \dfrac{\dot{W}}{2k^2}\dfrac{2}{3}\sigma_{xx}$

$= \dfrac{\dot{W}}{3k^2}\sigma_{xx}.$

Volume stays constant.

$\dot{W} - \dot{W}_{tot} = \sigma_{xx}\,\dot{\varepsilon}_{xx}$

#2

Total strain rate along path BD: elastic + plastic.

$$\dot{\varepsilon}_{xx} = \underbrace{\dot{\varepsilon}^{el}_{xx}} + \underbrace{\dot{\varepsilon}^{pl}_{xx}}$$

$$= \frac{\dot{\sigma}_{xx}}{E} + \frac{\dot{W}}{3k^2}\sigma_{xx}$$

plug in $\dot{W}$ expression:

$$\left(1-\frac{\sigma_{xx}^2}{3k^2}\right)\dot{\varepsilon}_{xx} = \frac{\dot{\sigma}_{xx}}{E}.$$

....algebra:

$$E\frac{\dot{\varepsilon}_{xx}}{\sqrt{3}k} = \frac{\dot{\sigma}_{xx}/\sqrt{3}k}{1-(\sigma_{xy}/\sqrt{3}k)^2}$$

Solution to this eqn.

$$E\frac{\varepsilon_{xx}(t)}{\sqrt{3}k} = \operatorname{atanh}\left(\frac{\sigma_{xx}(t)}{\sqrt{3}k}\right)$$

rewritten as:

$$\boxed{\frac{\sigma_{xx}(t)}{\sqrt{3}k} = \tanh\left(E\frac{\varepsilon_{xx}(t)}{\sqrt{3}k}\right)}$$

yield condition

$$\frac{1}{3}\sigma_{xx}^2 + \sigma_{xy}^2 = k^2.$$

Von Mises.

$$J_2 = \frac{1}{2}S_{ij}S_{ij}\cdots$$

Solve for σ_{xy}. along path BD

$$\frac{\sigma_{xy}}{k} = \sqrt{1-\left(\frac{\sigma_{xx}}{\sqrt{3}k}\right)^2}$$

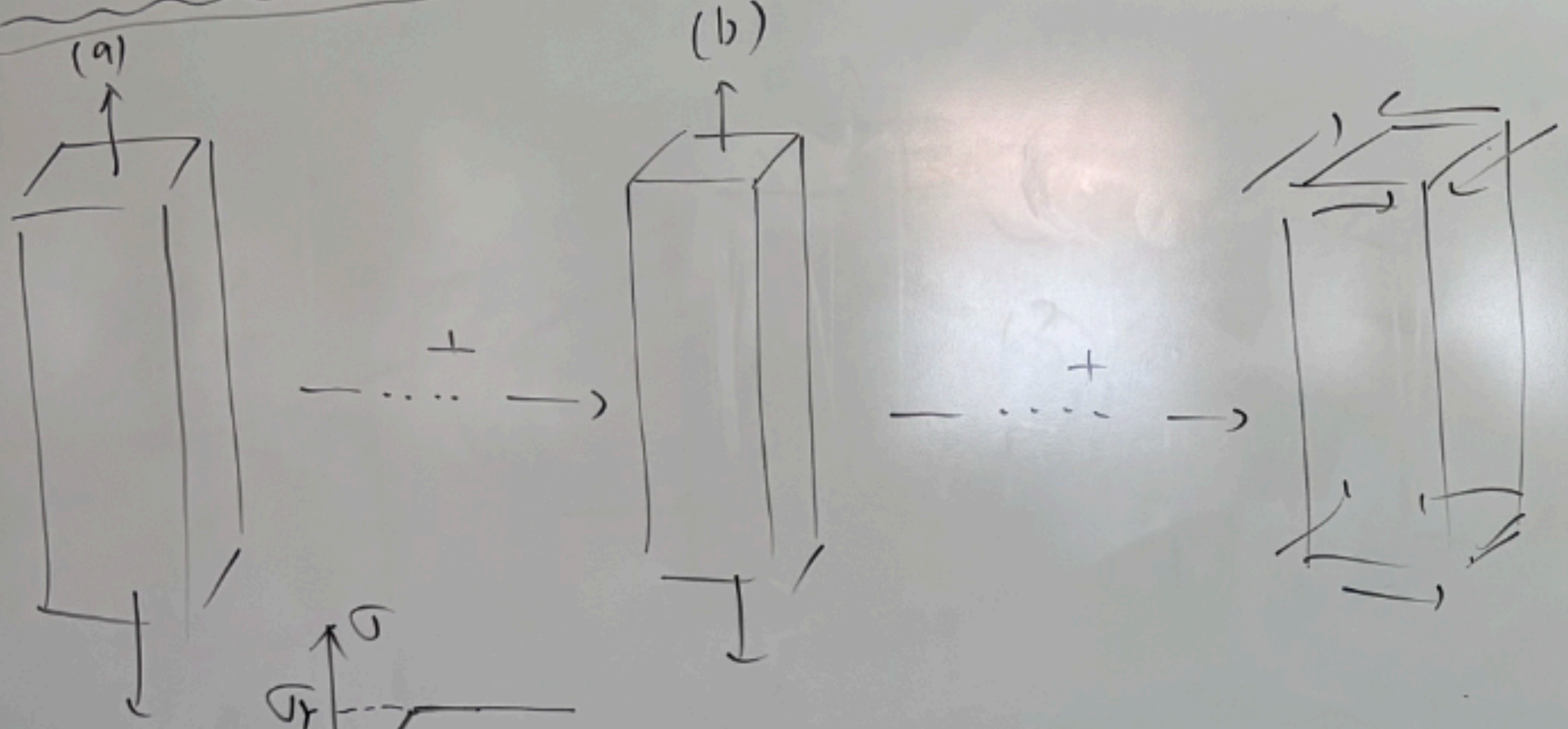
$$= \frac{1}{\cosh(E\varepsilon_{xx}(t)/\sqrt{3}k)}$$

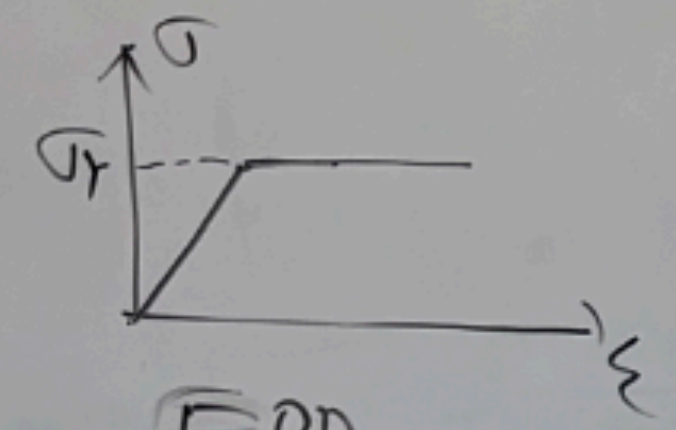
$$= \frac{1}{\cosh(\sqrt{3}\mu\varepsilon_{xx}(t)/k)}$$

#3


Scenario A
Wed.
OA → AD.
D.
σ_{xx}/k
1.93
1.12
1
0.76
0.46
0
A
1
2
$2\mu\varepsilon_{xy}/k$
$\sigma_{xx} = \sqrt{3}k$
$\sigma_{xy} = 0$.
$\sigma_{xx} = 1.12k$
$\sigma_{xy} = 0.76k$.
Scenario B
Tri.
OB → BD
1.173
$\sqrt{3}$
1.617
1.31
σ_{xy}/k
σ_{xx}/k
0.64
0.217
B
D.
$E\varepsilon_{xx}/\sqrt{3}k$
$\sigma_{xx} = 1.31k$
$\sigma_{xy} = 0.64k$
~~LINEAR ELASTIC~~
Scenario A
Scenario B
(a)
(b)
Wed.
O → A
A → D
Tri
O → B
B → D
ε_{xx}
$2\varepsilon_{xy}$
σ_{xx}
σ_{xy}
O
A
D.
t
$\gamma_{xy} = 2\varepsilon_{xy}$
B
σ_{xx}
σ_{xy}
L.E.

Extend Scenario A





EPP