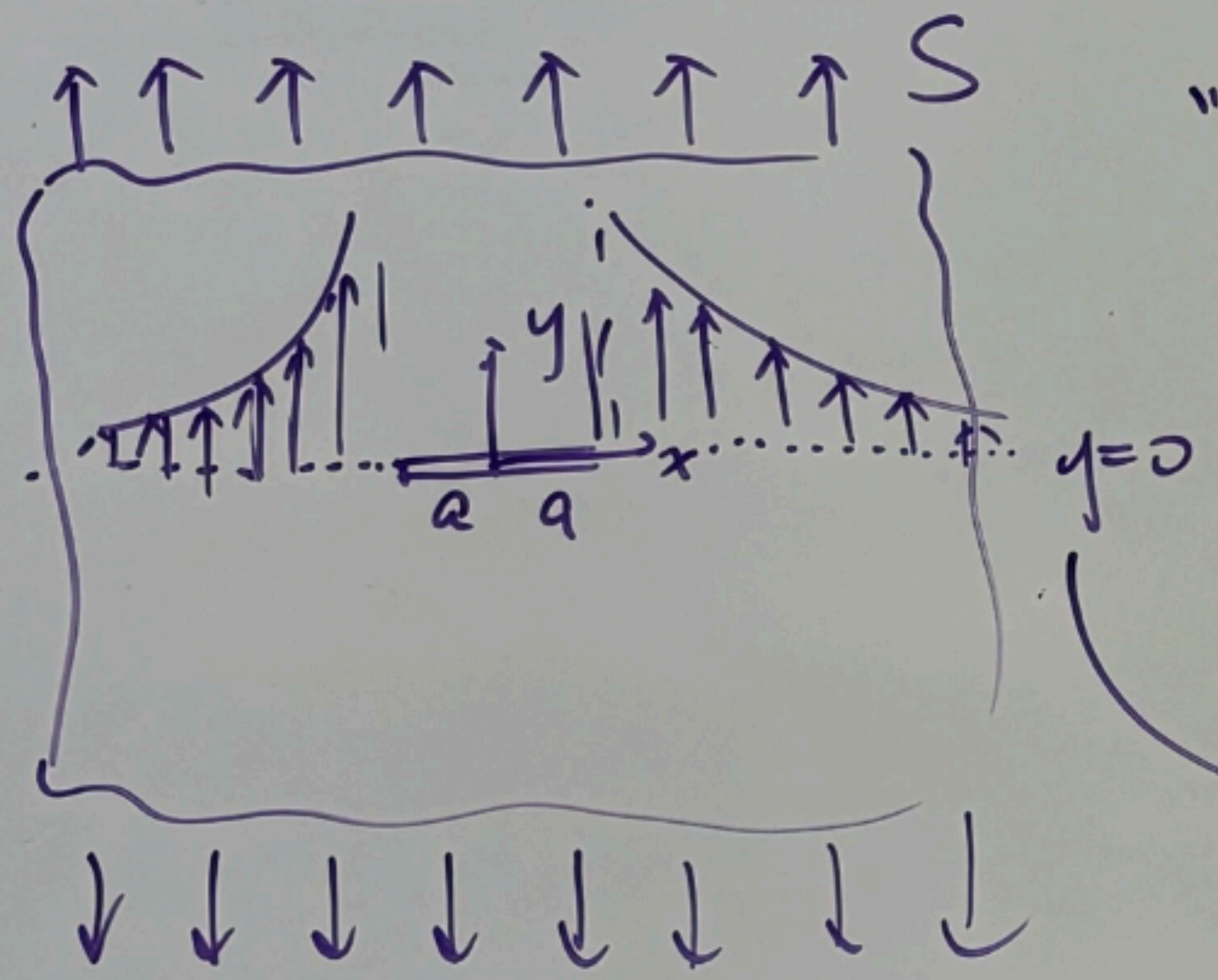
Problem Session

- fracture Mechanics

LEFM.



Slit-like crack.

"normal traction. dist. on crack"

$$p(x) = \frac{S(x)}{\sqrt{x^2-a^2}}.$$

@ $x, y=0$ $\sigma_{yy}(r) = \frac{K_I}{\sqrt{2\pi r}}$

Singular integral. equation.

$$d(x) = g(x) = \int_{-a}^{a} \frac{p(x')}{x-x'}\, dx'$$

Crack. opening. disp. (COD)

$$d(x) = \frac{2(1-\nu)}{\mu} S a \sqrt{1-\left(\frac{x}{a}\right)^2}.$$

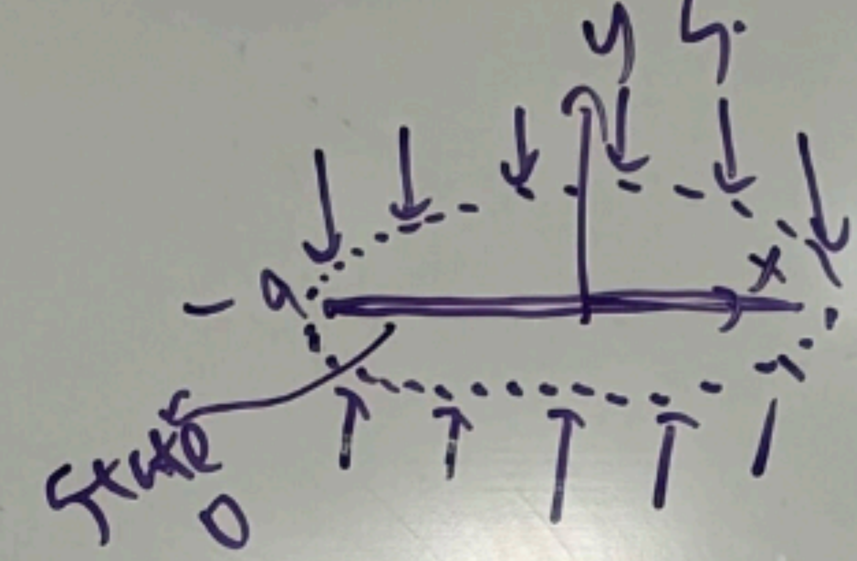
$-a \le x \le a$.

evolves in the direction of enthalpy.

$H = \dot{\hat{E}} - W_{cm.} = -\hat{E}$

* linear elastic solid.

no pre-existing internal stress



a. Applying S

State 1

↓

State 0

State 0.

↓

no crack

$\hat{E}_0$. H_0.

↳ $H_0 = -\bar{E}_0$.

State 1

↓

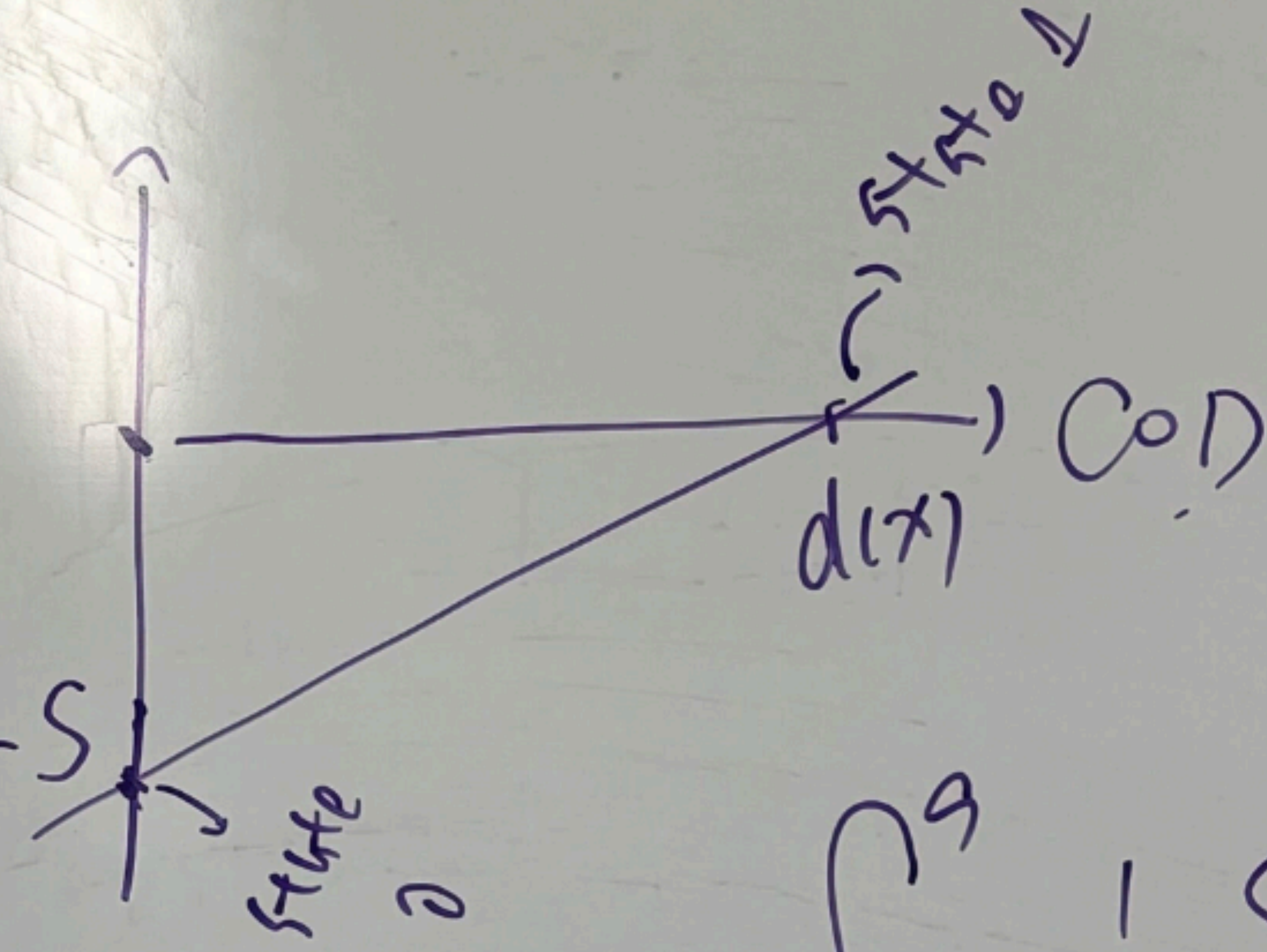
with crack & opened.

$\hat{E}_1$, H_1.

$\Delta H = -\Delta \dot{E}$. strain energy

↳ $\Delta E > 0$.

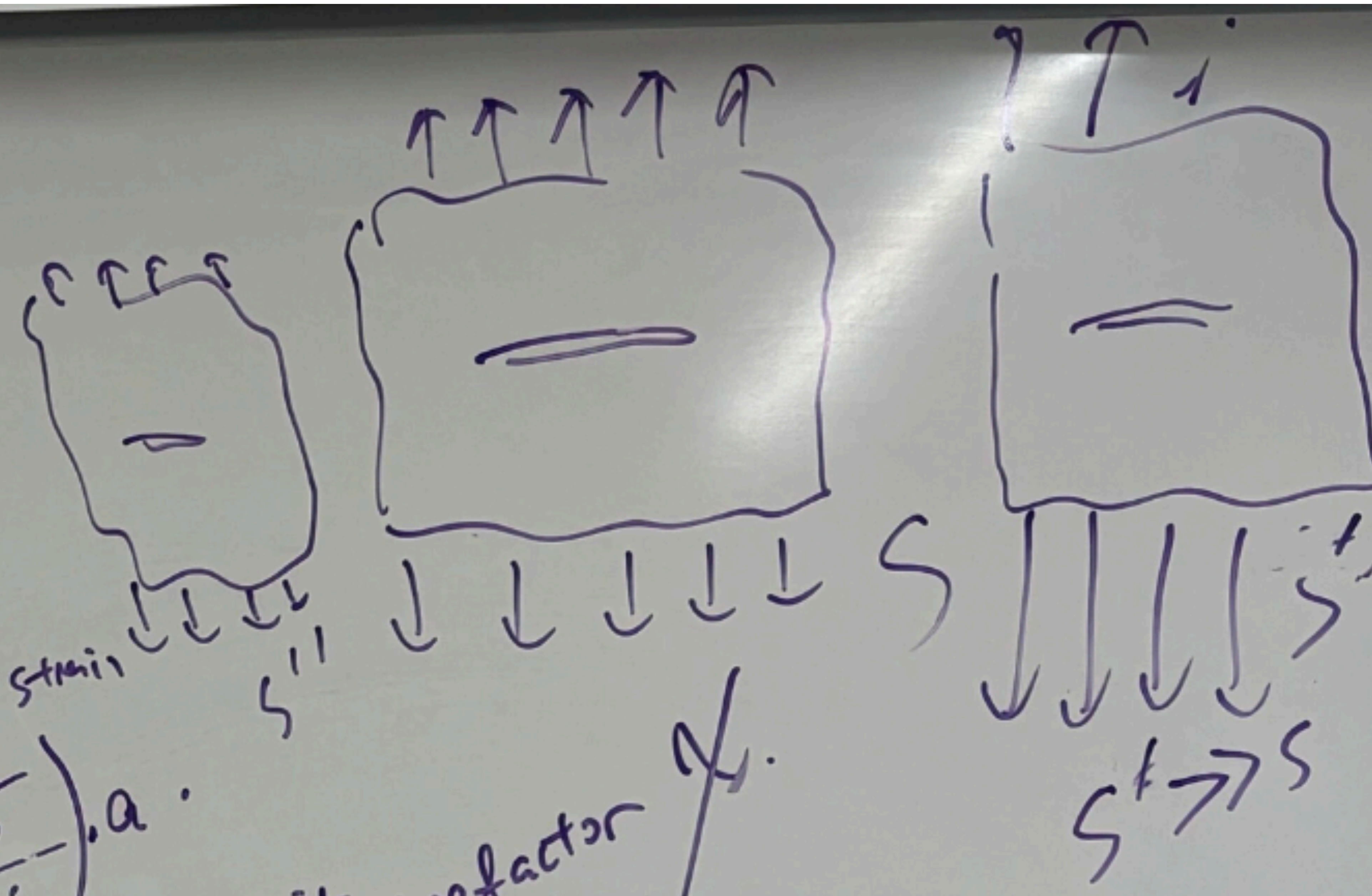
the internal energy of solid increasing in fracture.



$$W_{cev} = \int_{-a}^{a} -\frac{1}{2} S(x) \, dx \, \underbrace{d(x)}_{COD}$$

Griffith Criterion

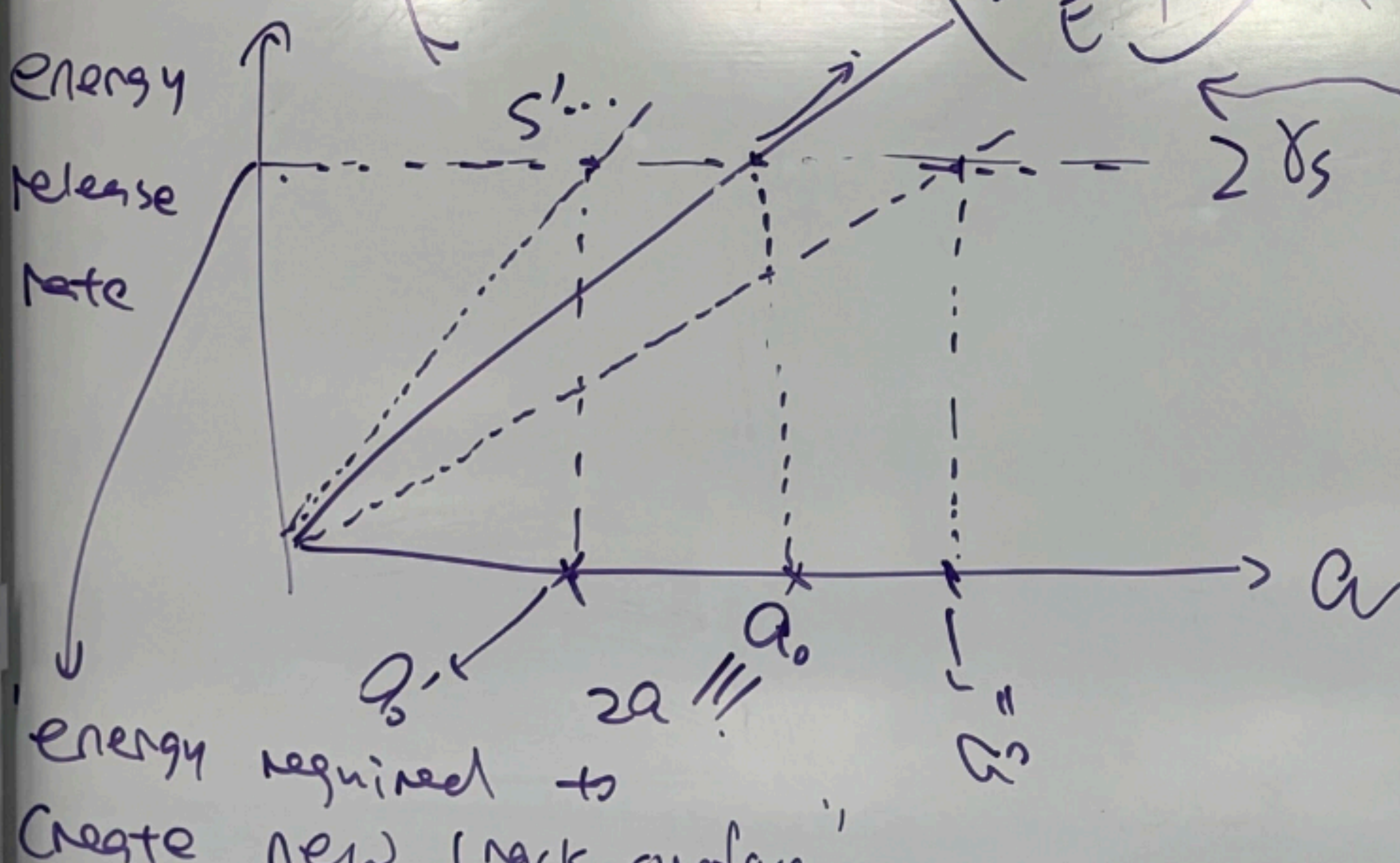
$$f_{el} = -\frac{\partial(\Delta H)}{\partial(\text{crack length})} = -\frac{\partial(\Delta H)}{\partial(2a)}$$



$$G.\ \left(f_{tot} = -\frac{\partial(\Delta H)}{\partial(2a)} - 2\gamma_s \right)$$

plane strain: $\left(\frac{\pi(1-\nu)S^2}{2\mu}\right) a$ → elastic energy prefactor χ

$\left(\frac{\pi S^2}{E'}\right) a$



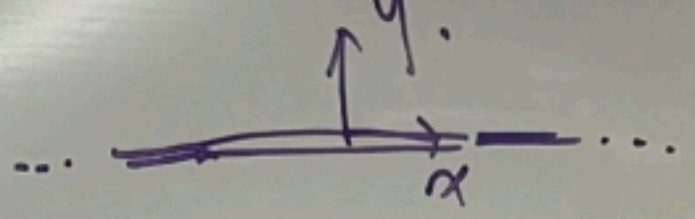
$$\Delta\Pi = -\chi a^2 + 2\gamma_s (2a)$$

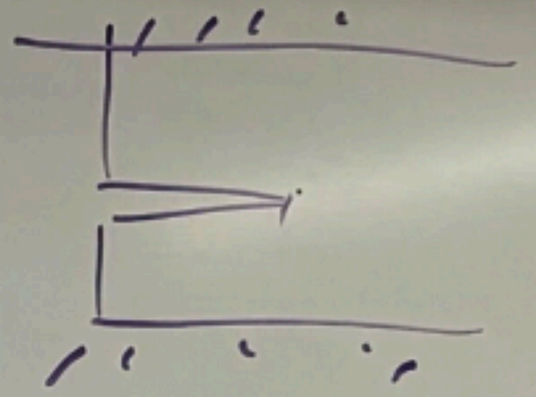
$$G(a) = -\frac{d\Pi}{d(2a)}$$

$$G(a) = \left(\frac{\pi S^2}{E'}\right) a$$

J - integral

$$J_i = \int_S (w n_i - T_j u_{j,i}) dS.$$



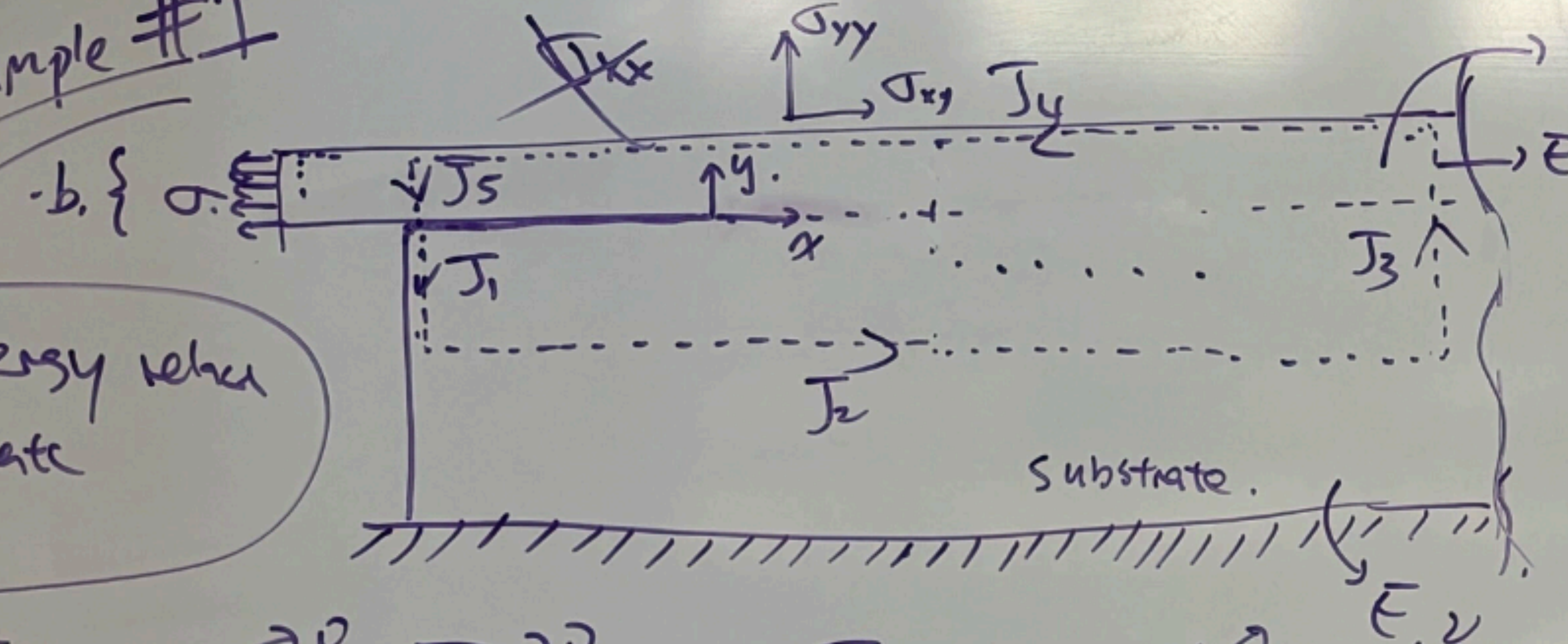


Rice ← Eshelby

1D
x-direction.

$$J_x = \int_\Gamma \overset{(1)}{(w\,dy)} - \overset{(2)}{\left(T \frac{\partial u}{\partial x} dS\right)}$$

Example #1



energy release rate

$J_1: \cancel{W}^{\to 0} \quad \cancel{T}^{\to 0}$

$J_2: W \cdot \cancel{dy} \quad T \cancel{\frac{\partial u_x}{\partial x}}, \cancel{\frac{\partial u_y}{\partial x}}$

$$J_x = \sum_{i=1}^{5} J_i$$

$J_3: \cancel{W} \quad \cancel{T}^{\to 0}$ "traction free"

$J_4: W \cdot \cancel{dy}, \quad \cancel{T}^{\to 0}$

plane strain.

$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E'}$$

$$J_5 \overset{(1)}{=} \int w\,dy = \int \frac{1}{2}\sigma_{ij}\varepsilon_{ij}\,dy.$$

$$w = \frac{1}{2}\sigma\varepsilon = \frac{\sigma^2}{2E'}$$

$$\overset{(1)}{\int_b^0} w\,dy = \frac{\sigma^2 b}{2E'}.$$

$G =$

$= \frac{\sigma^2 b}{2E'}$.

plane strain.

$\varepsilon_{xx} = \frac{\sigma_{xx}}{E'}$.

(2). $\int_b^0 T_x \frac{\partial u_x}{\partial x}\, dy$.

$T_x = \sigma_{xx} = \sigma$, $\frac{\partial u_x}{\partial x} = \varepsilon_{xx}$

$= \int_0^b -\sigma_{xx}\varepsilon_{xx}\, dy$

$= -\int_0^b \frac{\sigma^2}{E'}\, dy$

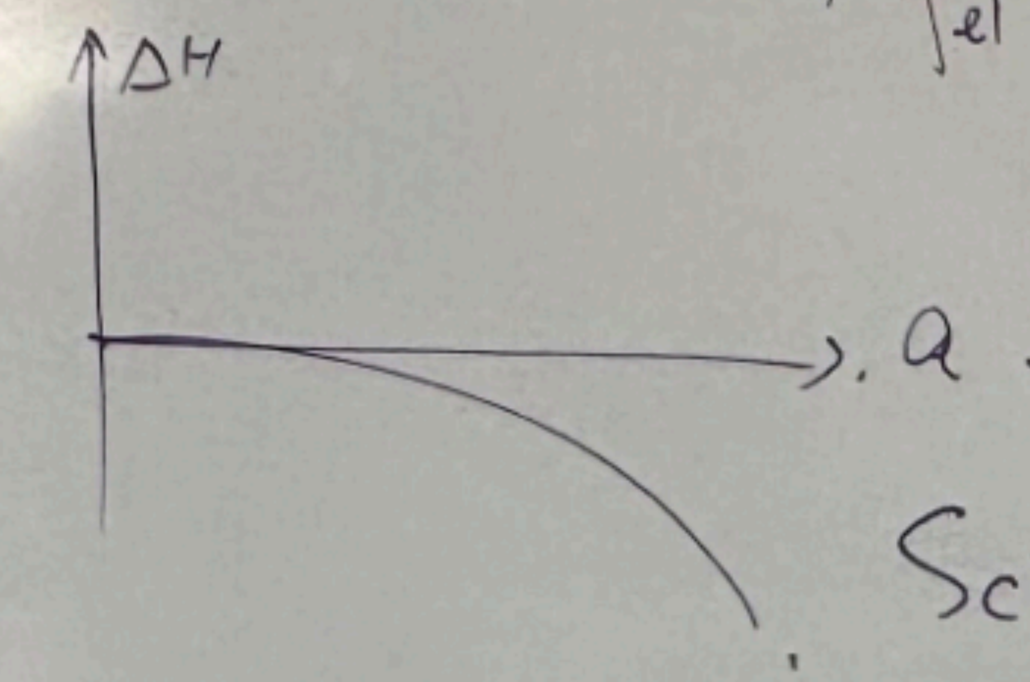
$= -\frac{\sigma^2 b}{E'}$

$G \equiv J_x - J_s = \frac{\sigma^2 b}{2E'} - \left(-\frac{\sigma^2 b}{E'}\right) = \frac{3\sigma^2 b}{2E'}$

"higher loading" $\rightarrow$ "steeper curve" $\rightarrow$ "shorter a_c" $\rightarrow$ "smaller critical crack length" $\leftarrow$ "crack is easier to grow".

$\Delta H = -\frac{(1-\nu)}{2\mu} S^2 \pi a^2$.

$\hookrightarrow f_{el} = \frac{\pi(1-\nu)S^2 a}{2\mu}$ ... (*)



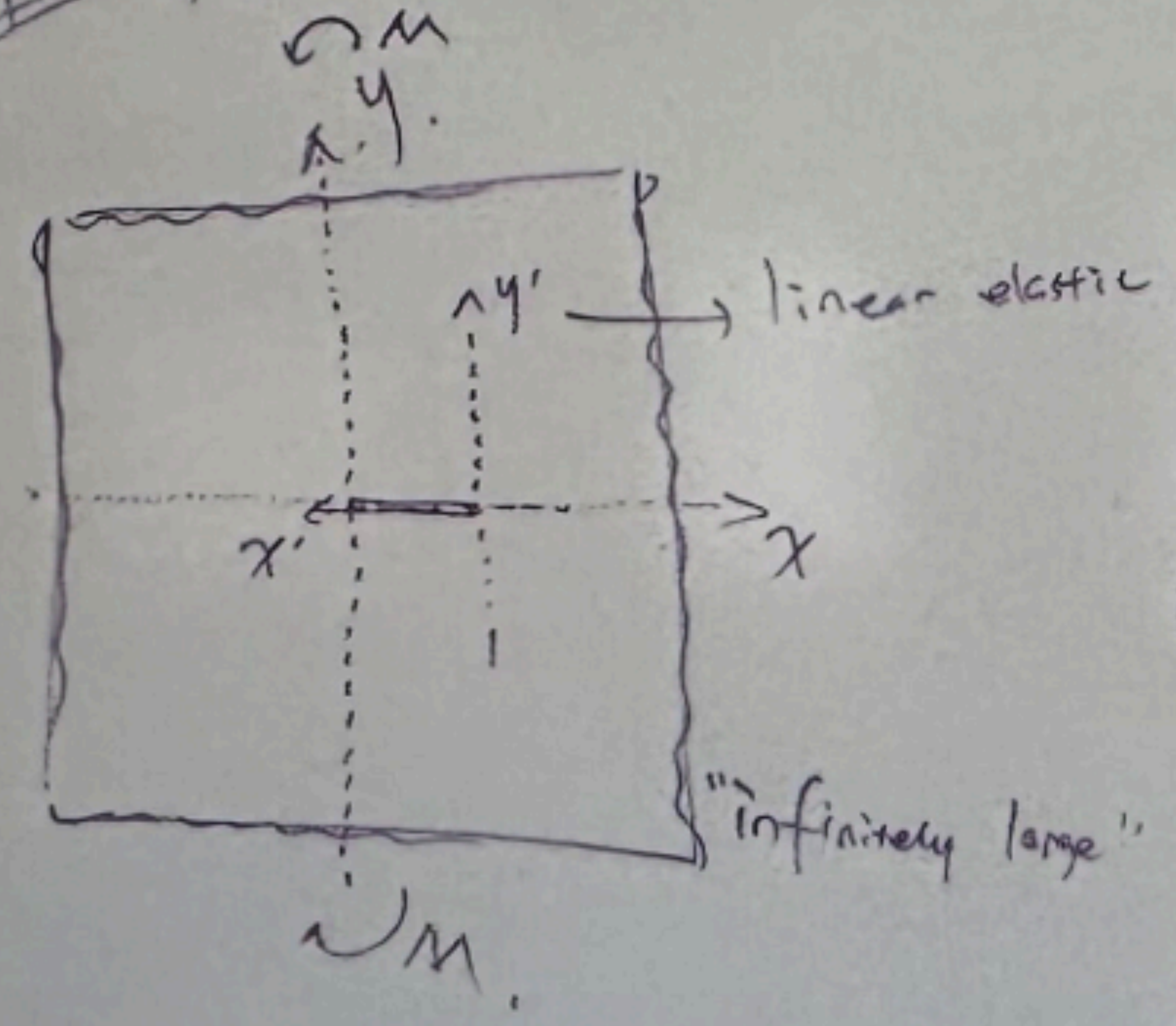
$f_{el} = 2\gamma_s$ ... (**)

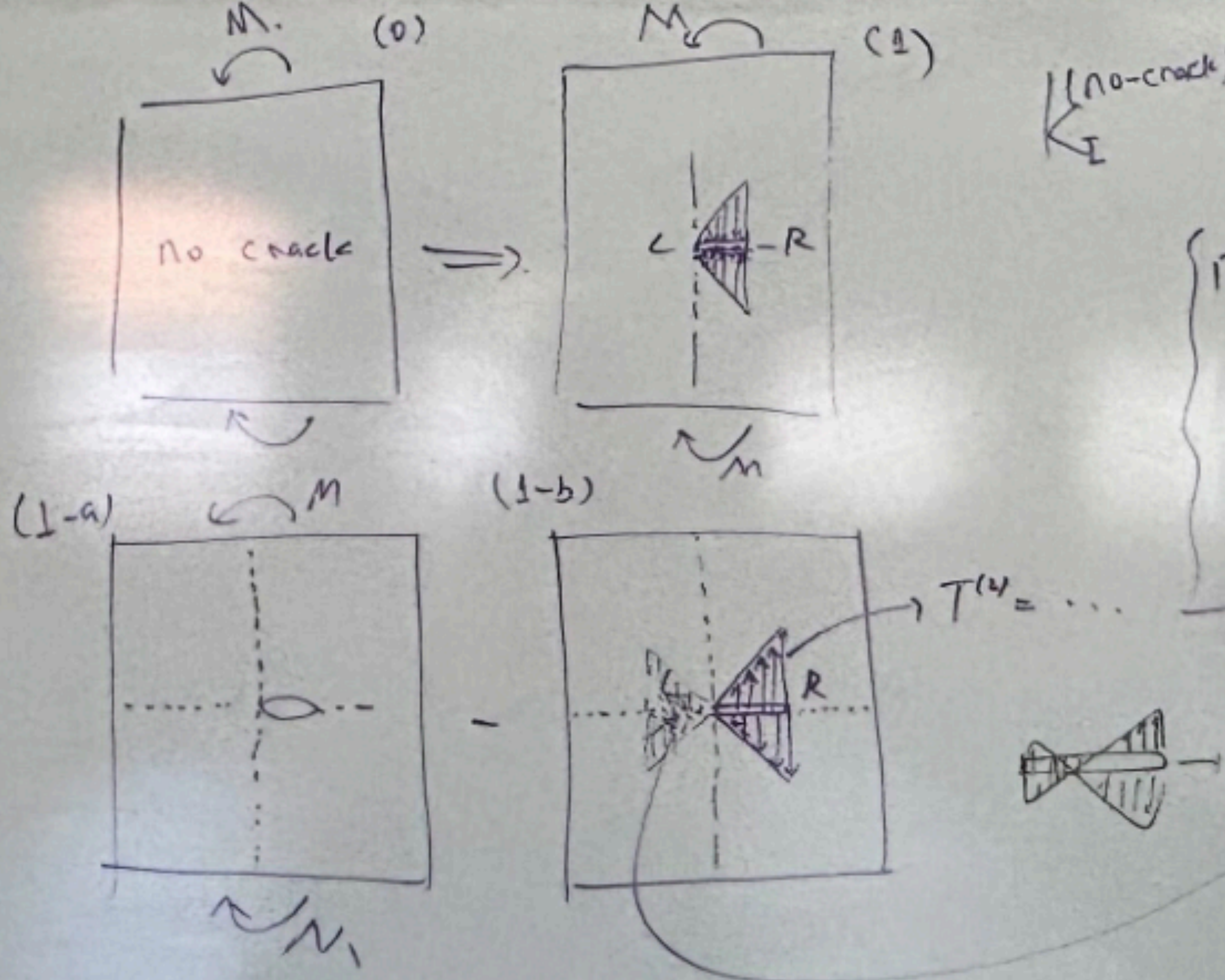
Subs. (*) into (**),

$S_c = \sqrt{\frac{8\mu\gamma_s}{\pi(1-\nu)(2a)}}$ "onset of unstable crack growth"

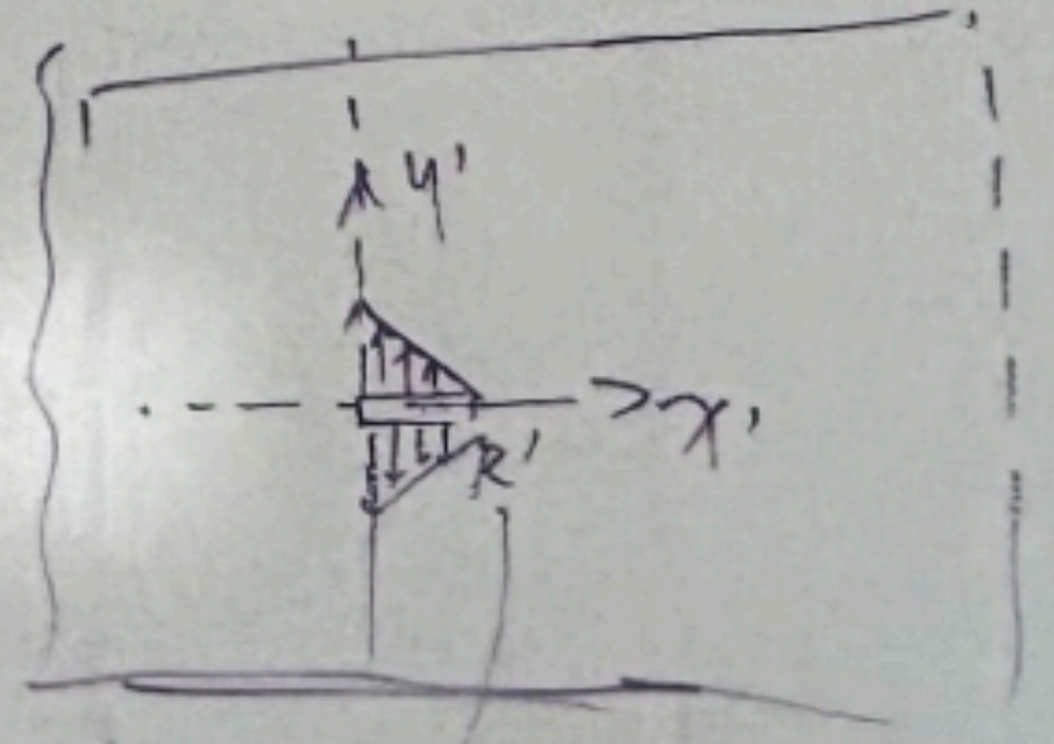
$2a_c = \frac{8\mu\gamma_s}{\pi(1-\nu)S^2}$

Example #2

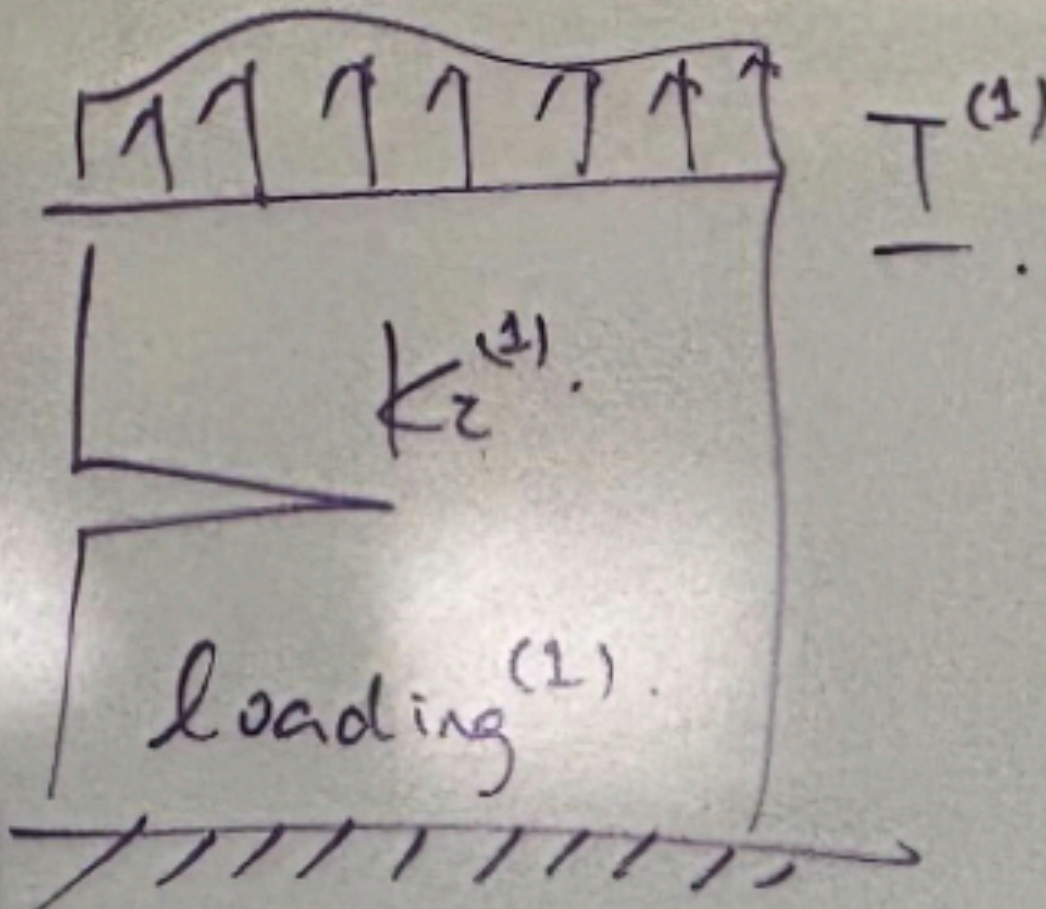


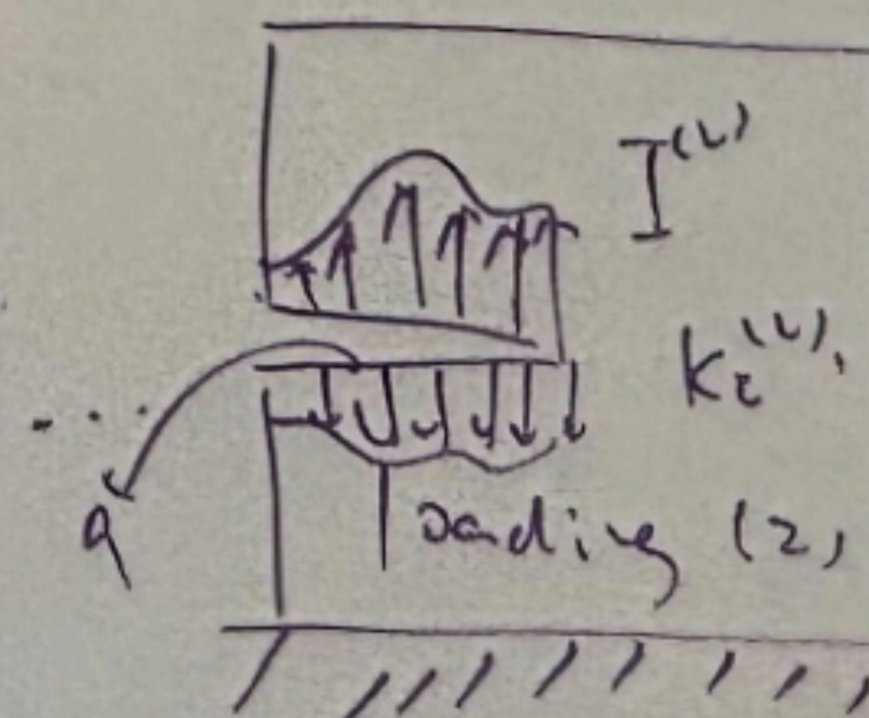


$$K_I^{(\text{no-crack})} = K_I^{(a)} - K_I^{(b)}.$$



Lecture notes. Rice (1973)





$$K_I^{(2)} \doteq \frac{E'}{2K_I^{(1)}} \int_{\Gamma} T_i^{(2)} \cdot \frac{\partial u_i^{(1)}}{\partial a} \, d\Gamma,$$

crack length.